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ENGINEERING EXPERIMENT STATION
BULLETIN SERIES No. 314

TESTS OF REINFORCED CONCRETE SLABS SUBJECTED TO CONCENTRATED LOADS

A REPORT OF AN INVESTIGATION

CONDUCTED BY

THE ENGINEERING EXPERIMENT STATION
UNIVERSITY OF ILLINOIS

IN COOPERATION WITH

THE UNITED STATES BUREAU OF PUBLIC ROADS

AND

THE ILLINOIS DIVISION OF HIGHWAYS

BY

FRANK E. RICHART

AND

RALPH W. KLUGE

JUNE 20, 1939

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CONTENTS

	PAGE
I. Introduction	7
1. Object of Tests	7
2. Acknowledgment	8
II. Tests of Long Rectangular Slabs Simply Supported on the Two Long Edges	9
3. Outline of Tests	9
4. Description of Test Slabs	11
5. Loading Apparatus	14
6. Dynamometers	15
7. Reaction Measuring Devices	15
8. Strain and Deflection Measurements	19
9. Testing Procedure	23
10. Data of Preliminary Control Tests	26
11. Results of Concentrated Load Tests on Uncracked Slabs	27
12. Concentrated Load Tests Producing Initial Cracks in Slabs	39
13. Concentrated Load Tests Producing General Cracking	45
14. Concentrated Load Tests to Failure	46
III. Tests of Square Slabs to Determine Effect of Size and Shape of Load Bearing Area	51
15. Object of Tests	51
16. Outline of Tests	53
17. Description of Test Slabs	53
18. Description of Tests	54
19. Results of Tests of Series A	56
20. Results of Tests of Series B	59
21. Effect of Type of Reinforcement on Manner of Final Failure	62
22. Comparison of Ultimate Loads	64
IV. Discussion and Conclusions	66
23. General Discussion	66
24. Conclusions	68
Appendix A. The Effect of Flexibility of the Reaction Weighing Rings on the Measured Reactions for the Test Slab	72

LIST OF FIGURES

NO.		PAGE
1.	Location of Load Points, Gage Lines, and Deflection Points on Test Slabs	11
2.	General Arrangement of Slab and Test Apparatus	12
3.	View of Slab No. 1 During Test	13
4.	Dynamometers Used in Slab Tests	16
5.	Reaction Weighing Ring	17
6.	View of Reaction Rings in Slab Test	18
7.	Brown Bubble Extensometer	20
8.	Bubble Extensometer Used on Under Side of Slab	21
9.	Graphic Strain Gage	22
10.	Sketch Showing Slab and Coordinates Used in Analysis	28
11.	Deflections of Slab on Sections Parallel to x Axis, with Load at Mid-span	31
12.	Deflections of Slab on Sections Parallel to x Axis, with Load at Quarter Point of Span	32
13.	Reactions Along One Edge of Slab, with Load at a Distance of ¼-, ½- and ¾-Span from the Support	34
14.	Values of Weighted Strain, $N\epsilon$, in Slab Due to Load at Mid-span	36
15.	Values of Weighted Strain, $N\epsilon$, in Slab Due to Load at Quarter Point of Span	37
16.	Uncorrected Load-Strain Curves Used to Detect Formation of Cracks	40
17.	Values of Concrete Strains Due to Large Loads	42
18.	Values of Concrete Strains Due to Large Loads	43
19.	Values of Steel Strains Due to Large Loads	44
20.	Location of Points of Application of Loads Producing Local Failure	46
21.	Top View of Test Slab at Point of Failure	48
22.	Bottom View of Slab Beneath Load Point After Failure	48
23.	Bottom View of Slab Showing Conical Surface of Fracture	49
24.	Profiles of Sections of Failure, Slab No. 2	50
25.	View Showing Method of Loading Test Slabs, Series B	54
26.	Variation in Strain in Concrete Along Longitudinal Center Line	57
27.	Variation of Strain Along Center Reinforcing Bar	58
28.	Variation of Strain Along Center Reinforcing Bar	59
29.	Load-Strain Curves for Reinforcing Bar Directly Under Load	60
30.	View of Test Slab After Failure	61
31.	Relations Between Diameter of Loading Disk and Yield Point and Ultimate Loads	63
32.	View of Slabs of Supplementary Series After Failure	65
33.	Slab Having Flexible Support Under One Edge	73

LIST OF TABLES

NO.		PAGE
1.	Schedule of Loadings in Tests of Large Slabs	10
2.	Properties of Concrete Used in Test Slabs	14
3.	Summary of Loads at Which Initial Cracks Formed in Test Slabs	41
4.	Arrangement of Loads and Strain Measurements on Test Slabs	52
5.	Summary of Tests to Determine Effect of Loading Area	62
6.	Maximum Reactions at A Per Unit Length	74

TESTS OF REINFORCED CONCRETE SLABS SUBJECTED TO CONCENTRATED LOADS

I. Introduction

1. *Object of Tests.*—The tests reported in this bulletin form a part of an investigation of reinforced concrete slabs subjected to concentrated loads, which was undertaken to secure information needed for the more effective design of highway bridge floor slabs. The present inadequacy of information on the subject is doubtless due largely to the mathematical difficulties met in an analytical approach to the subject. For example, until recently methods of calculating the effect of flexibility of supporting beams, and of interaction of slab and beams were lacking or very limited, and even though available, were generally too complex for general use in design. On the other hand, an experimental approach to the question without first having available an analytical treatment with which to rationalize and generalize the test results, would be both expensive and ineffective. In the present program, therefore, the plan was to make a series of tests, employing slab types for which analytical solutions were readily available, and to determine what differences exist between the measured and the calculated quantities. A good agreement between even a limited number of test and analytical values will do much to justify the use of the theory in an untold number of other cases, which do not depart too widely from the conditions present in the tests.

Another purpose of the tests was to determine physical quantities and relations which are not available from analysis. The tests of Chapter III, for example, were made to determine the effect of size and shape of the bearing area to which load is applied. In the well-known analysis by Westergaard* the effect of a concentrated load is modified by assuming the load uniformly distributed over a small circular area. To supplement this, the tests were made with the load applied through circular disks, rings and pairs of disks, of various sizes and arrangements. The tests were expected to indicate whether variation in area and distribution of loading from vehicle tires in service is of any importance with regard to slab design.

Another somewhat indirect object of the tests is concerned with the reliability, or consistency, of slab behavior. As might be judged from analytical procedure, such quantities as the reaction between

*H. M. Westergaard, "Computation of Stresses in Bridge Slabs Due to Wheel Loads," Public Roads, Vol. 11, No. 1, March, 1930.

slab and supporting member are extremely sensitive to small variations in deflection of the support. The tests, then, should indicate the importance of such variations, which might be caused in service by temperature and moisture changes, poor initial contact of bearing surfaces, etc., upon essential properties of the slab.

In general, the tests may be considered as having two distinct phases. The first involves the behavior of the uncracked test slab, which might be expected to give close agreement with the mathematical analysis, since the properties of the materials of the slab up to this stage approach the conditions of perfect elasticity assumed in the analysis. The second phase covers the action of the slab after cracks have formed, wherein close agreement between measured and calculated quantities is less to be expected. Although in this second stage the behavior of the slab may depart considerably from that of the elastic structure, the test results are of prime importance in indicating the ultimate safety of the structure in service.

It may be noted that results of previous portions of this investigation are already available in two bulletins giving analytical solutions or methods of analysis for various types of slab; also in a bulletin describing tests of plaster of paris slab models made to verify certain of the design assumptions.*

The tests covered herein fall into two distinct groups. One group was confined to two large rectangular slabs, simply supported on the two long edges and subjected to a single concentrated load, which was applied successively at a number of points on the slab. The other group of tests was made on eighteen slabs, with the width equal to the span, simply supported on two edges and subjected to a single central load applied through distributing plates of varying shapes and areas. Since the two groups of tests are somewhat dissimilar, they will be described separately in Chapters II and III, respectively.

2. *Acknowledgment.*—This investigation is being conducted in the Engineering Experiment Station with the cooperation of the United States Bureau of Public Roads and the Illinois Division of Highways. The project is under the administrative direction of Dean M. L. Enger, Director of the Engineering Experiment Station, Professor F. B. Seely, Head of the Department of Theoretical and Applied

*V. P. Jensen, "Solutions for Certain Rectangular Slabs Continuous Over Flexible Beams," Bul. 303, Eng. Expt. Sta., Univ. of Ill. 1938.

N. M. Newmark, "A Distribution Procedure for the Analysis of Slabs Continuous Over Flexible Beams," Bul. 304, Eng. Expt. Sta., Univ. of Ill. 1938.

N. M. Newmark, "Tests of Plaster Model Slabs Subjected to Concentrated Loads," Bul. 313, Eng. Expt. Sta., Univ. of Ill. 1939.

Mechanics, and Professor W. C. Huntington, Head of the Department of Civil Engineering.

The program of the investigation is guided by an Advisory Committee, having the following personnel:

Representing the U. S. Bureau of Public Roads:
E. F. Kelley, Chief, Division of Tests
A. L. Gemeny, Senior Structural Engineer
Representing the Illinois Division of Highways:
Ernst Lieberman, Chief Highway Engineer
Alfred Benesch, Engineer of Grade Separations
Representing the University of Illinois:
F. E. Richart, Research Professor of Engineering Materials
N. M. Newmark, Research Assistant Professor of Civil Engineering
Consultants to the Committee, from University of Illinois:
W. M. Wilson, Research Professor of Structural Engineering
T. C. Shedd, Professor of Structural Engineering
Hardy Cross, Professor of Structural Engineering (prior to July, 1937)

Especial acknowledgment is made to Dr. Newmark and to Dr. V. P. Jensen, Special Research Assistant Professor of Theoretical and Applied Mechanics, for continued assistance in the planning and conduct of the tests reported herein, and particularly for contributing analytical studies involved in Section 11 and Appendix A.

Credit is also due to Henry A. Lepper, Jr., Research Graduate Assistant, who made detailed numerical computations for the investigation.

II. Tests of Long Rectangular Slabs Simply Supported on the Two Long Edges

3. *Outline of Tests.*—The tests were made on two large slabs of identical design, each simply supported on the two long edges, and having a transverse width equal to three times the span. With these proportions of the slab it is known that a load at the middle produces about the same maximum moments as would a load applied on a slab of equal span but infinite width. The schedule of Table 1 shows the magnitude and position of the concentrated loads that were applied to the two slabs, and Fig. 1 shows the position of gage lines and deflection points for loads below 16 000 lb. For loads of

TABLE 1
SCHEDULE OF LOADINGS IN TESTS OF LARGE SLABS

Stage of Test	Applied Load lb.	Load Point	Measurements Taken	Apparatus and Instruments
Slabs uncracked	4 000 8 000	A, B, C (See Fig. 1)	Complete set of strains and deflections. Reactions measured on slab No. 1	Brown bubble gage. Graphic strain gage. Ames dial deflectometer. Reaction rings. 20 000-lb. dynamometer
Appearance of initial cracks	8 000 to 12 000	A, B, C, D, E, F, G, H, I	Uncorrected strains	Bubble gage used to detect initial cracks
General development of cracks	8 000 16 000 24 000 32 000 40 000 48 000 56 000	A, B, C, D, E, F, G, H, I	Strains in concrete and steel	Reaction rings removed. Berry and Whittemore strain gage (slab 1). Graphic gage (slab 2). 125 000-lb. dynamometer
Loads to localized failure*	60 400 to 88 000	(See Fig. 20)	Strains in concrete and steel	Berry and Whittemore strain gage (slab 1). Graphic gage (slab 2). 125 000-lb. dynamometer

*Failures generally occurred by punching hole through slab after yield point of steel was reached.

16 000 lb. and more, some new load points and gage lines were used to permit direct readings on the reinforcing steel.

Live load reactions were measured along the middle half of the length of one of the supports of the first large slab, at nineteen separate points spaced symmetrically about the middle of the support. In this way about 95 per cent of the total live load reaction was measured.

Strains were measured in the concrete over a 4-in. or 6½-in. gage length on both the top and bottom of the slab at various points along and across the x and y axes, as indicated in Fig. 1. Deflections were measured over the entire top surface of the slab, at 10-in. intervals in both directions.

The tests of the slabs proceeded in 4 distinct stages: (1) Tests of the uncracked slab, at loads generally under 8000 lb., with very careful and complete measurement of strains, deflections and reactions; (2) tests at loads which produced initial cracking with reaction measuring rings replaced by a rigid steel rail support, and with strain measurements used to detect first cracking of the slab; (3) tests at loads producing general development of cracks in the slab; and (4) tests to failure of the slab.

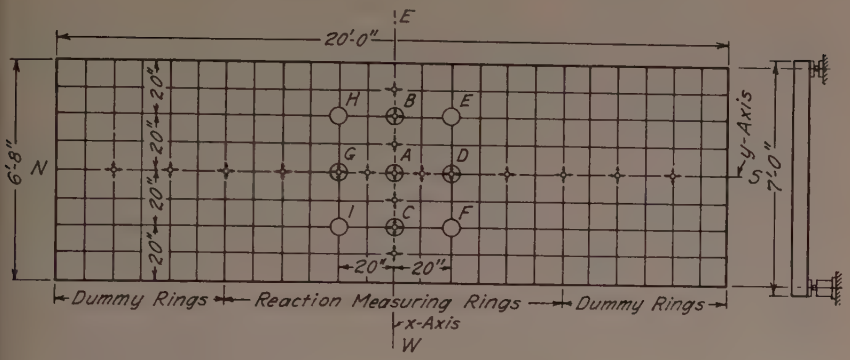


FIG. 1. LOCATION OF LOAD POINTS, GAGE LINES, AND DEFLECTION POINTS ON TEST SLABS

Position of loads and gage lines modified at later stages of test

4. *Description of Test Slabs.*—Each of the test slabs was 20 ft. wide, 7 ft. long, and 6½ in. thick, with a simple span of 6 ft. 8 in. between centers of supports. One edge of the slab was supported on a row of cold-rolled steel blocks 4 in. wide, 24 in. long, and 1¼ in. thick, anchored to the slab and placed end to end to form a segmental bearing plate extending the entire 20-ft. width of the slab. Each block rested on a roller, 1½ in. in diameter and 24 in. long, the rollers in turn resting on a cold-rolled steel bearing plate grouted to the concrete pier. The other long edge of the slab was supported in two ways. In the first test it was supported on cold-rolled steel blocks, 4 in. by 6 in. and 1½ in. thick. In the lower surface of each block a hemispherical recess was provided to fit a 1-in. steel ball, through which the reaction was transmitted to the reaction measuring ring beneath. Figures 2 and 3 show the arrangement of the slab and testing equipment. A different scheme was used in the later stages of the test of slab No. 1 and throughout the test of slab No. 2, wherein the edge of the slab was supported by a steel rail in place of the row of reaction rings.

Both slabs were reinforced with ½-in. deformed square bars of intermediate grade steel, spaced 4 in. center to center in both directions, with the bars in the direction of the span anchored at the ends

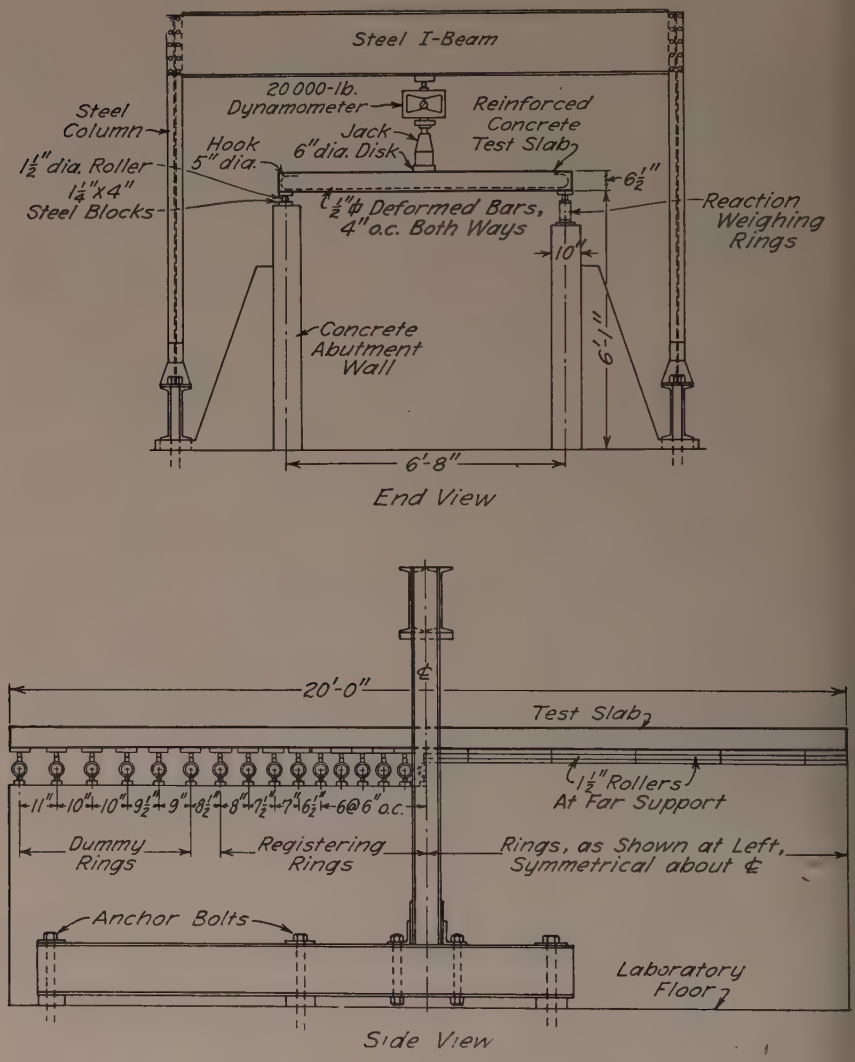


Fig. 2. General Arrangement of Slab and Test Apparatus

by hooks of 5-in. diameter. The effective depth to the bars in the direction of the span was 5½ in. Properties of the steel used for these slabs were determined from tests of several bars. The average values were

Tensile yield point, 45 500 lb. per sq. in.

Ultimate tensile strength, 72 800 lb. per sq. in.

FIG. 3. VIEW OF SLAB NO. 1 DURING TEST

The proportions of the slab concrete were 1 : 3½ : 4¼ by weight, and a water cement ratio of 0.95 to 1.0, by volume, was used. The materials employed were a standard portland cement and Wabash river torpedo sand and gravel. The fineness modulus of the sand was about 3.0; that of the gravel varied, since the material in the first slab was graded from ¼ to 1½ in.; that in the second, from ¼ to 1 in. The concrete was mixed 4 minutes in a "Wonder" batch mixer of ⅕ cu. yd. capacity. Reinforced concrete control beams, each roughly equivalent to a 12-in. strip of the large slab, were made, principally for determination of elastic constants for the reinforced slab. A group of 6-in. by 12-in. control cylinders were also made and cured with the slabs; properties of these cylinders are given in Table 2.

TABLE 2

PROPERTIES OF CONCRETE USED IN TEST SLABS 1 AND 2

Properties of 6- by 12-in. cylinders. Concrete proportions, 1 : 3½ : 4¼, by weight. Materials, standard portland cement, Wabash river torpedo sand, and gravel.

Cylinder No.	Age at Test, days	Compressive Strength lb. per sq. in.	Initial Modulus of Elasticity thousands of lb. per sq. in.	Remarks
1	29	4000	3500	Made with slab No. 1
3	29	3590	3350	$w/c = 1.0$
7	29	3370	3750	
10	29	3890	3650	
Average	..	3710	3560	
2	263	3600	4350	Made with slab No. 1
4	263	3880	3870	$w/c = 1.0$
5	263	4080	3980	
6	263	3790	3400	
8	263	3460	4090	
9	263	3360	3900	
Average	...	3705	3930	
4	59	4240	4250	Made with slab No. 2
7	59	4600	4750	$w/c = 0.95$
8	59	4500	3830	
11	59	4200	3960	
Average	..	4385	4200	
3	91	4090	3240	Made with slab No. 2
9	91	4440	4450	$w/c = 0.95$
13	91	3690	3350	
Average	..	4070	3660	

The slabs were poured with bearing blocks, rollers, and other supporting units in place, for the purpose of having all parts of the supports in bearing at the beginning of the test. They were left in the forms with the exposed surface covered with wet burlap for a period of one week. The forms were then removed and the slab allowed to dry in the laboratory air for 28 days before any load was applied.

5. *Loading Apparatus.*—As shown in Fig. 2, load was applied to the test slab by means of a screw jack, reacting against a structural steel frame, which in turn was anchored to the heavily reinforced concrete floor of the laboratory. (The floor, 16 in. thick, contains 2½-in. bolt sockets at 6 ft. intervals, each capable of resisting a 50 000-lb. upward pull.) The loading structure was designed to withstand safely a jack reaction of 150 000 lb. The jack reaction was measured by means of two dynamometers, one

of 20 000-lb. capacity, used for the lighter loadings, and one of 125 000-lb. capacity, for the loads to failure. The position of the steel loading frame was adjustable, so as to permit the use of the various load points of Fig. 1.

The load was transmitted to the slab through a cast-iron disk, 6 in. in diameter and 2 in. thick, bedded onto the slab with plaster of paris.

6. *Dynamometers.*—Two dynamometers for measuring the load applied to the slab were designed and built for the purpose. A view of these dynamometers is given in Fig. 4.

The dynamometer of 20 000-lb. capacity was cut from a solid plate of nickel steel. It consists of a closed rigid frame, with load applied at the middle of a $9\frac{3}{4}$-in. span. Deflections are measured by a $\frac{1}{10\,000}$ in. micrometer dial gage, attached by means of a saddle to the bottom member of the frame. Calibration curves indicate that the instrument will indicate the load with a precision of ± 50 lb. at all loads up to capacity. At the design load of 16 000 lb. on the slab, the tolerance amounts to ± 0.3 per cent.

The dynamometer of 125 000-lb. capacity consists of a heat-treated steel ring, having an outside diameter of 10.5 in., an inside diameter of 8 in., and a width of 5 in. The ring, of chrome-vanadium steel (S.A.E. 6145), was first turned out from a forging, then bearing block and dial yokes were attached by fillet welding, and the whole ring was heat-treated to produce a yield strength of about 190 000 lb. per sq. in. Deflections of the ring are indicated by a $\frac{1}{10\,000}$-in. micrometer dial gage. Calibration curves indicate that the precision of the instrument throughout its range is ± 175 lb.

Both dynamometers were loaded through a $1\frac{3}{4}$ in. steel ball.

7. *Reaction Measuring Devices.*—As previously noted, nineteen reaction measuring instruments were used to determine the distribution of reactions along one supported edge of the slab. It was required of these instruments that they indicate the load accurately and still sustain a minimum of vertical deformation, in order to maintain the support as nearly rigid as possible, since the usual analysis assumes perfect rigidity. A large number of devices were built and tested before one was chosen for the purpose.

The devices investigated included (a) friction disk devices,* in which the resistance to rotation of the disk should be proportional

*Such devices have given satisfactory service, when lower pressures were used. See Huntington, W. C. and Luetzelschwab, E. C., "New Type of Pressure Cell for Granular Material," Eng. News-Record, Vol. 116, No. 1, Jan. 2, 1936.

(a)

(b)

FIG. 4. DYNAMOMETERS USED IN SLAB TESTS

to the normal load, (b) friction ribbon devices,* similar in principle to the friction disk, (c) elastic weighing rings with carbon pile deflection measurement, and (d) elastic weighing rings, with multiplication lever and dial micrometer for deflection measurement. In class (a),

*M. G. Spangler, "The Distribution of Shearing Stresses in Concrete Floor Slabs under Concentrated Loads," Bul. 126, Iowa Eng. Expt. Sta., April, 1936.

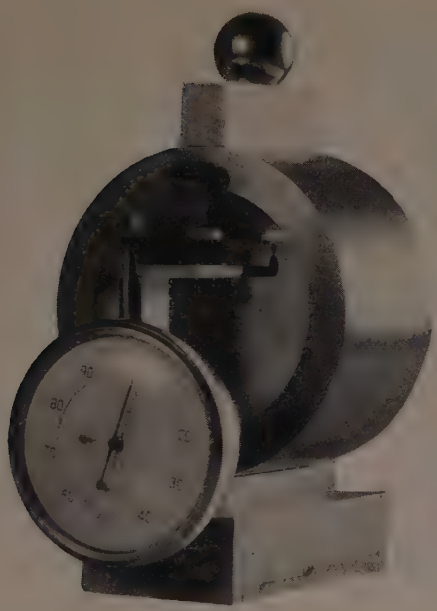


FIG. 5. REACTION WEIGHING RING

the torque required to rotate a disk under load between two bearing surfaces was determined for the following combinations of bearing material: steel and bronze, steel and brass, steel and graphite, steel and "oilite" bearing metal, steel and hard wood, cast iron and bronze, carbon and bronze, "Gadke" composition and bronze, and tool steel and bronze. In class (b), friction ribbons or tapes of stainless steel or brass were pulled at a uniform rate, between bearing blocks of bronze or stainless steel. In class (c), the principle of the double-stack carbon resistor telemeter was used. One stack of carbon disks was placed so as to indicate horizontal ring deflections, the other, vertical deflections. In class (d), a small steel ring was used with (1) wedge extensometers for measuring deflections, and (2) a multiplying lever and dial micrometer for measuring deflections.

It was desired to measure reactions with a precision of ± 1 per cent and at the same time confine the maximum vertical deformation of the measuring device to 0.001 in. Furthermore, the size of the

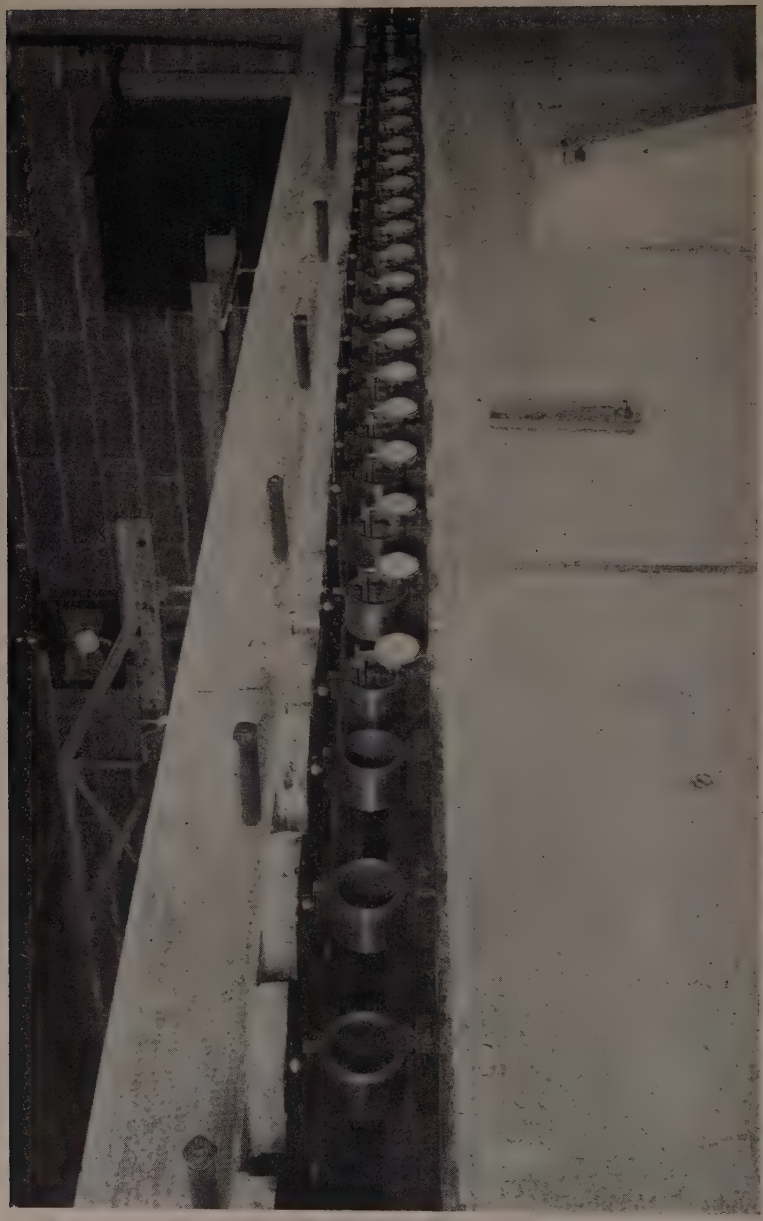


FIG. 6. VIEW OF REACTION RINGS IN SLAB TEST

devices was limited by the close spacing at which they were to be used, so that fairly high bearing pressures resulted. Under these restrictions, the friction devices did not show a satisfactory accuracy or reliability, evidently due to the scoring and galling of the surfaces in contact. Neither did the carbon pile telemeters used in the reaction rings produce the desired accuracy of reading.

The reaction ring selected for the slab tests is shown in Fig. 5. It was 2½ in. long, with outside and inside diameters of 3⅞ and 2⅞ in., respectively, and was designed to deflect vertically a maximum of 0.001 in. under a load of 1000 lb. Deformation of the ring was indicated by an Ames micrometer dial graduated to 0.0001 in., actuated by a lever of approximately 10 to 1 multiplication ratio. Thus one dial division represents about 0.00001 in. deflection of the ring, corresponding to a 10-lb. change in load. The lever mechanism employed integral fulcrums; the lever, fulcrums, supporting blocks, and dial support were made in one piece by milling the unit from a ½ in. plate of nickel or tool steel. To insure positive action the unit was welded to the ring at four points. The base of the instrument contained a carefully machined movable wedge and a tapered follow block upon which the ring was supported, thus permitting a very fine vertical adjustment of the ring to secure equalization of dead load at the beginning of a test. Load was applied to the ring through a 1-in. steel ball resting in a spherical recess in the top surface of the ring. Each ring was carefully calibrated up to a load of 1000 lb. A comparison of several calibrations for a single ring indicated that the error in load for any individual reading should not exceed 20 lb.

Figure 6 shows the arrangement of the rings under the test slab. The spacing of the rings was variable, being smallest in the vicinity of the load, in order to equalize the ring deflections. Non-indicating or "dummy" rings were used in the outer fourths of the span width, where the reactions were very small, in order to maintain some degree of uniformity in the stiffness of the support as a whole.

8. *Strain and Deflection Measurements.*—For measuring strains in the test slab at low loads, the ordinary portable strain gage was not sufficiently sensitive, and two types of fixed gage or extensometer were used. The first, devised by R. L. Brown, of the Department of Theoretical and Applied Mechanics, is called the "bubble" gage, since it utilized sensitive level bubbles for measuring rotations.*

*Rex L. Brown, "A Level-Bubble Strain Gage." A.S.T.M. Bulletin, No. 93, p. 16, August, 1938.

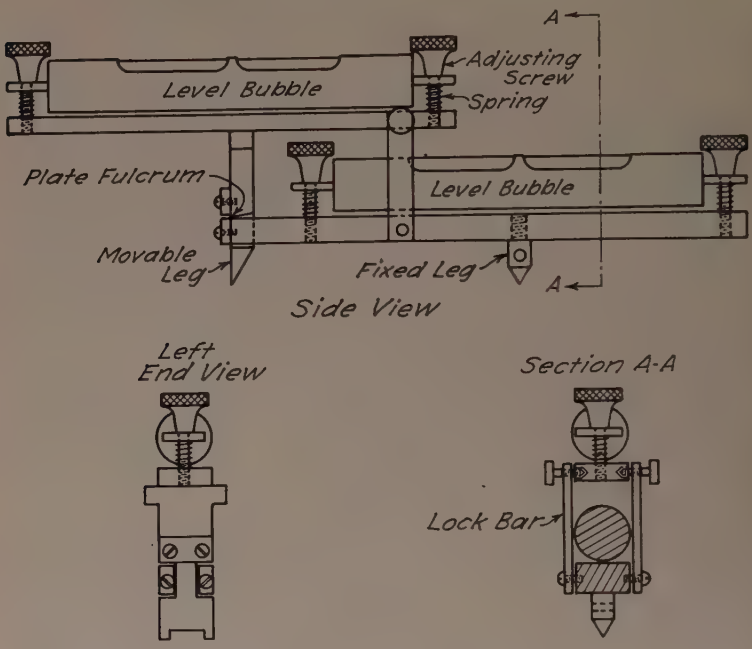


Fig. 7. Brown Bubble Extensometer

Figure 7 illustrates the instrument, which, with a 5-second bubble, has a multiplication ratio of about 3300 to 1. The instrument, of 4 in. gage length, has one fixed and one movable leg. The latter is attached to the body of the instrument by a plate fulcrum; a change in length of the gage line causes a rotation of the leg and of the bubble attached thereto. Readings were taken by means of the graduations engraved on the bubble vial. One division corresponded to a strain of 0.000007 in. per in., and the instrument could be read to one fifth of a division. Since strains in a slab were generally accompanied by deflections which would rotate the entire instrument, a second bubble was attached to the body of the instrument to determine overall rotations which could be used to correct the strain readings. Another method of making the correction was to repeat the test with the ends of the instrument reversed through 180 deg. The average of the two readings was thus free from the effect of overall rotations of the member.

The design of the gage is such as to permit a considerable range of adjustment of the bubble, though the range of the instrument is necessarily limited. The bubble is so mounted that its weight is

FIG. 8. BUBBLE EXTENSOMETER USED ON UNDER SIDE OF SLAB

well balanced. When the gage is used on the top of the slab, it is self-supporting, and requires only a light tap over the knife-edge supports to make them engage the steel or brass gage plugs attached to the concrete slab. To use the instrument on the lower side of a slab, the level bubble was inverted and a mirror attached to facilitate readings, as shown in Fig. 8. The instrument was held against the slab under constant pressure by a steel spring.

The second type of gage, used on slab No. 2, is known as the "Graphic Strain Gage,"* developed by the Bureau of Public Roads. The instrument, shown in Fig. 9, consists essentially of a frame, of 6½-in. gage length, carrying a bell-crank lever with a multiplication ratio of 60 to 1, by means of which the magnified strains are recorded on a smoked glass slide. The conical points of the instrument are adjusted so as to bear firmly against metal plugs embedded in the concrete and projecting above the surface. Deformation within the gage line produces motion of the sliding pin, *D*, at the right terminal which bears upon a knife edge, *E*, which is one extremity of the bell-crank. The distance from this knife edge to the plate fulcrum or

*L. W. Teller, "An Improved Recording Strain Gage." Public Roads, V. 14, No. 10, p. 189, December, 1933.

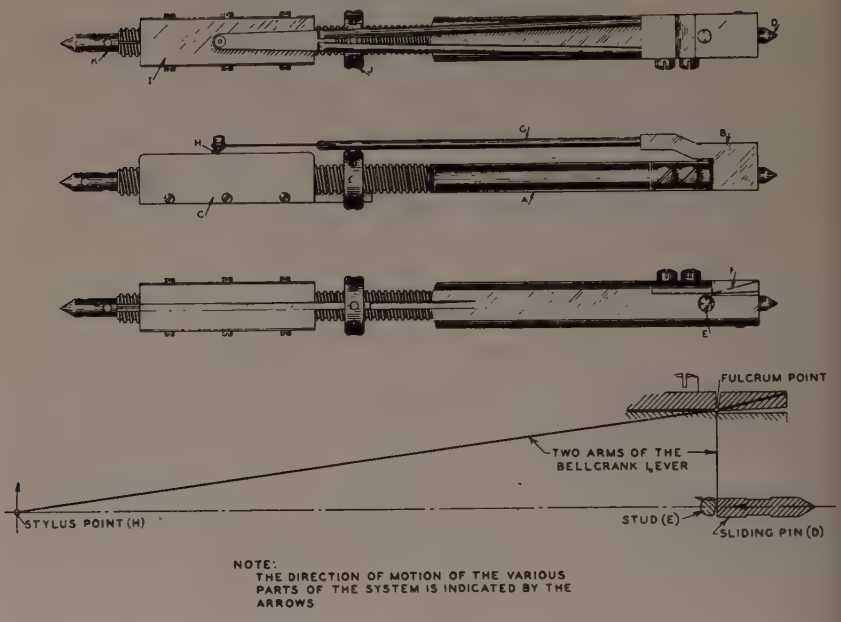

FULCRUM POINT
TWO ARMS OF THE BELLCRANK LEVER
STYLUS POINT (H)
STUD (E)
SLIDING PIN (D)
NOTE:
THE DIRECTION OF MOTION OF THE VARIOUS PARTS OF THE SYSTEM IS INDICATED BY THE ARROWS

FIG. 9. GRAPHIC STRAIN GAGE

spring hinge, *F*, constitutes the short arm of the bell-crank; the other arm consists of two metal rods, *G*, carrying a stylus, *H*, which is in contact with a smoked glass plate, *I*. A capstan nut on the threaded body of the instrument, *A*, provides a convenient method of moving the record slide longitudinally between successive strain measurements. With this adjustment, twenty or more readings can be recorded on one glass slide. Obviously, the final evaluation and recording of the strains is an office procedure, accomplished by means of a microscope with a micrometer stage, generally after the completion of the test.

One feature of the instrument deserves mention. To minimize error due to temperature changes the body of the instrument from end to end was made of "Invar," a nickel-steel alloy. Further, to compensate even the thermal expansion of the Invar, a bimetallic stylus arm was used in which the differential in coefficient of expansion was utilized to reduce the error to a negligible amount. The instruments were carefully calibrated before use. It is evident that the sensitiveness of the instrument depends upon the mechanical ratio of the instrument, the magnification employed in the microscope used, and the sharpness of the record. The accuracy of strain

determination depends principally upon the accuracy with which the path of the stylus can be read; this will be greatest if the stylus is kept sharp and the smoke film is kept as thin as possible. Experience with these gages indicates that a precision of strain measurement of 0.000005 in. per in. can be obtained without difficulty.

The bubble gage was used in the first of the large slab tests, the graphic gage in the second: in addition, all strains after cracks had formed in the slabs were measured by means of 5-in. Berry and Whittemore strain gages, capable of producing a precision of 0.00001 in. per in., or better.

Deflections of the slabs were measured by means of steel frames or bridges, fixed in position, each carrying a battery of $\frac{1}{10\ 000}$-in. Ames dials, 10 in. apart, and spanning the slab as shown in Fig. 3. Two instruments were made of steel bar-joists, stiff yet fairly light, supported on two legs at one end and on one at the other. In use these frames rested on small steel ball bearings partially embedded in the concrete directly over the slab supports. For the center-line or x-axis deflections, a third deflection frame was required to straddle loading disk and jack. This was made of steel angles, with crossbars carrying the micrometer dials.

To provide metallic contacts between dial spindles and the slab, small sections of cold-rolled steel plate were attached to the surface of the slab at 10-in. intervals in both the x and y directions. With this precaution, there was no difficulty in repeating readings to a precision of 0.0002 in.

9. *Testing Procedure—Slab No. 1.*—Since the tests of the two large slabs were not identical in all respects, the procedure followed with each of the individual slabs will be recorded. Test slab No. 1 was cast in place; however, early readings on the reaction weighing rings indicated that the bearing between the slab and rings was not satisfactory at certain places along the support. As a result the slab was raised slightly and the bearing plates above the rings were grouted in place with a very thin layer of neat cement. After this had hardened, the rings were brought to essentially equal bearing under the dead load of the slab by means of the adjusting wedges and screws provided for this purpose in the ring bases.

As a preliminary to the main series of tests on the slab, it was felt desirable to secure data on the stiffness properties of the slab. For this purpose, auxiliary test beams had been cast with the slab, having about the same span, thickness, and reinforcement as the large slab, but having a width of only one foot. It was thought that from these control beams, values of deflections and the corre-

sponding values of EI for use in computing the quantity $N = EI/1-\mu^2$ could be determined for use in connection with the main slab. In this expression, E is the modulus of elasticity, I is the moment of inertia per unit of width of slab, and μ is Poisson's ratio. However, these control beams differed appreciably in thickness from the main slab, and this introduced some question as to the proper method of adjusting the value EI. After some study, it was decided to test the slab as a simple slab with a continuous line load applied at mid-span along the y axis. Such a loading would produce flexure of the sort to be found in the control beam, and should give a constant deflection at mid-span along the entire width of the slab. This test was carried out by loading the slab over a strip about 8 in. wide by means of cast-iron weights and cement in sacks which were piled up to produce a loading of about 400 lb. per lin. ft. The values of the deflections and the resulting quantity N were used in computing the deflections to be expected under concentrated loading in the remainder of the test.

The subsequent loading procedure was as follows: A single concentrated load was applied successively at the middle of the slab and at the one-quarter points of the span along the center line of the slab normal to the supports. These load points are indicated as points A, B and C of Fig. 1. Loads of 4000 and 8000 lb. were first applied in this way. The three tests included readings of the reaction rings for the load placed on the center line of the slab as previously noted, and also with it placed six inches on either side of the center line, thus distributing the same total reaction over a slightly different portion of the slab, and over different weighing rings in each test. There were thus nine different load points employed so far as the reaction measurements were concerned. The testing procedure was as follows. A set of reaction measurements were taken with only the weight of the slab, loading jack, and dynamometer resting on the rings; for convenience these readings are termed zero load readings. With load increments of 4000 and 8000 lb. applied to the slab a complete set of strain and deflection measurements was also taken. In each case the load was released and the operation was repeated as a check.

The strains taken were measured on the concrete on both the upper and the lower surfaces of the test slab at thirteen points along the y axis spaced at 20-inch intervals, except for two points which were 10 inches each side of the center; also at 7 points along the x axis spaced at 10-inch intervals. The location of these gage lines is shown in Fig. 1. To provide suitable bearings for the bubble

gage points, small brass plates were attached to the test slab by means of plaster of Paris and the gage was mounted on these plates. It may be noted that the procedure followed in using this gage over most of the test was to take a set of readings with the gage placed in one position, then to release the load, reverse the position of the gage, and take a second set of readings. The check readings on identical loads were required to agree within one fifth of a division of the bubble scale, corresponding to a precision of 0.0000014 in. per in. of strain.

The measurements of deflection were carried out in a manner similar to that used for measuring strains and reactions. With the deflection bridge or frame in position, a set of readings was taken at zero load, then others with the 4000 and 8000 lb. increments of load in place. The deflection bridge was then removed from the slab and replaced and a series of check observations was made. These readings were required to check within 0.0002 in.

Following the loads of 4000 and 8000 lb. a further series of loads was applied at points A to I (see sketch in Fig. 1) for the purpose of detecting the opening of the first crack at each of these points under the load. It happened that the first load in this series was applied at point C; a load of 12 000 lb. evidently produced a crack, as indicated by the strain measurements, and after considerable search the crack was located by means of a magnifying glass. After this the loads were applied more slowly and with extreme care; cracks were discovered at loads varying from 8000 to 13 000 lb. The initial cracks were all very small, and may be reasonably assumed not to be influenced by cracks at points twenty inches or more distant.

Following this series of loadings the reaction rings were removed to avoid their being overloaded, and were replaced by an A.R.E.A. 129.6-lb. railroad rail which was grouted into place to form a very solid rigid support for the slab. After this had been done a load of 12 000 lb. was again applied, and a complete set of strain readings was taken for comparison with readings taken at this load with the reaction rings in use. However, no significant difference in the two sets of strains was observed.

Further loads were applied to determine the stresses in the reinforcement after cracking had become fairly general. The loads were applied in increments of 8000 lb. up to 48 000 lb., and in some cases to 56 000 lb. Strain measurements were taken with Berry and Whittemore gages on the top surface, and on reinforcing bars on the bottom, which were exposed for the purpose at this stage of the test.

The slab was finally loaded to failure. By applying the load at points some distance apart, three distinct localized failures were obtained. From the magnitude of the different loads, and from the appearance of the slab in the proximity of each point of failure, it is believed that the second and third failures were unaffected by the previous local failure.

Slab No. 2.—The procedure with slab No. 2 differed somewhat from that used with slab No. 1, principally because in this case the reaction rings were not used to support the slab. Instead, the slab was supported on a section of steel rail such as that used in the later stages of the test of slab No. 1. The slab was cast in position on the supports with one edge supported by the steel rail and the other supported on steel rollers as in the first test.

As before, a preliminary series of tests was made to determine the elastic properties of the slab. This was done by applying a line load at mid-span along the entire y axis of the slab. This in effect used the test slab as a simple beam with center loading, and a quantity, $N = EI/1 - \mu^2$, was determined for later computations in connection with the concentrated loading.

Concentrated loads were applied successively at points A, B and C as indicated in Fig. 1, and careful measurements of strains and deflections were taken. In this test the graphic strain gage was used for strain measurements instead of the bubble gage. Strains were taken at points indicated in Fig. 1, and loads of 2000, 4000, 6000, 8000 and 10 000 lb. were applied successively.

Following this series of loads with careful strain measurements a further series of loads was applied to determine the load required to produce the first crack. Generally this load varied from 8000 to 13 000 lb., and was applied at eight different points on the test slab. This series of tests was followed by load applications in increments of 8000 lb. up to 48 000 lb., and in some cases to 56 000 lb., with strain measurements at each load.

Finally, loads were applied which caused failure of the slab. As in the test of slab No. 1, several local failures were produced, by loading the slab at points some distance apart. Judging from the maximum loads and the type of fracture, these failures were independent of each other.

10. *Data of Preliminary Control Tests.*—It has been noted that control beams were made with the test slabs for the purpose of determining stiffness and other properties. However, those made

with slab No. 1 were inadvertently made of a different depth from that of the slab, while those made with slab No. 2, although having the correct depth, showed abnormally low stiffnesses. Hence, tests were made on the slabs themselves, by means of a line load across the 20-ft. width at mid-span, and the results of these tests are believed to be much more reliable than any tests of companion control beams. A representative strip in the middle portion of the slab may be considered as a beam with a concentrated load at the center. The measured deflections of such strips from the slab tests have been used to determine values of the quantity, N, by use of the equation

$$N = \frac{Ps^3}{48f}$$

wherein P = load per unit width of slab
s = span
f = deflection at mid-span.

The values of N found experimentally were 112 000 000 in.^{2} lb. per in. of width for slab No. 1 and 125 000 000 in.^{2} lb. per in. for slab No. 2. If the value of Poisson's ratio is 0.15, these values of N correspond to values of EI of 109 000 000 and 122 000 000 in. lb. Computing I from a concrete section equivalent to the sum of the concrete and steel areas, and taking account of the fact that the actual average overall depths of slabs No. 1 and No. 2 were 6.56 and 6.63 in., respectively, gives values of E for the two slabs of 4 300 000 and 4 700 000 lb. per sq. in., respectively. While these values are somewhat higher than values found from tests of the control compression cylinders made with the slabs, the values appear to be reasonable ones, for the quality of the concrete used. It should be noted that the tests were extended over a period of several weeks, so that even though each slab was allowed to air-dry for 28 days before any tests were made, there was chance for some variation in E during the period in which each slab was tested.

11. *Results of Concentrated Load Tests on Uncracked Slabs.*— Values of the measured strains, deflections, and reactions from the slab tests may be studied and an estimate of their reliability may be made by comparing them with values computed from the theory of elastic slabs. In the present case, while the analysis of a rectangular slab having a width three times the span is available, the computations for a slab of infinite width were very much simpler to make and apply almost equally well to the test slab. That is, the error in

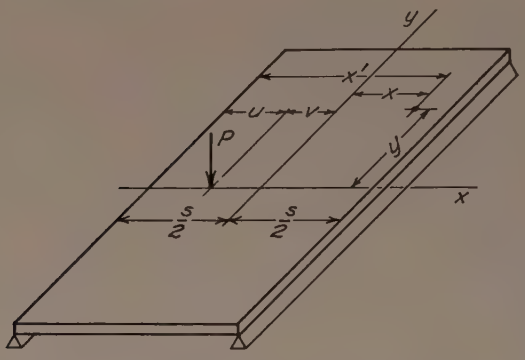


FIG. 10. SKETCH SHOWING SLAB AND COORDINATES USED IN ANALYSIS

determining the values of maximum strains and deflections by the latter method is known to be less than 1 per cent, and may be considered negligible, for slabs having a width-span ratio of 3. Hence the equations applicable to the slab of infinite width were used here.

The theoretical deflections, reactions and strains presented in this section were computed from equations available in the literature.* In stating the equation corresponding to Equation (1), Westergaard placed the origin of coördinates at the left edge of the slab. In the following, the origin of coördinates will be taken at the center of the slab, as shown in Fig. 10. Attention is called to the fact that while x is measured from the y axis, as usual, the distance x' is measured from the left edge of the slab. The dimensions u, v, and s are defined in Fig. 10.

The downward deflection w of the slab shown in Fig. 10 is given by Equation (1)

$$w = \frac{Ps^2}{2\pi^3 N} \sum_{1,2,3,\cdots}^{n} \frac{1}{n^3} \left(1 + \frac{n\pi y}{s}\right) e^{\frac{-n\pi y}{s}} \sin \frac{n\pi u}{s} \sin \frac{n\pi x'}{s} \qquad (1)$$

The upward reaction R at the left edge of the slab (along the line $x = -s/2$, Fig. 10) is given by Equation (2)

*See, for example: A. Nadai, "Die elastischen Platten," Julius Springer, Berlin, 1925; H. M. Westergaard, "Computation of Stresses in Bridge Slabs Due to Wheel Loads," Public Roads, Vol. 11, No. 1, March, 1930.

$$R = \frac{(3-\mu)P}{4s} \frac{\sin \frac{\pi u}{s}}{\cosh \frac{\pi y}{s} - \cos \frac{\pi u}{s}} \left[1 - \frac{1-\mu}{3-\mu} \frac{\frac{\pi y}{s} \sinh \frac{\pi y}{s}}{\cosh \frac{\pi y}{s} - \cos \frac{\pi u}{s}} \right] \quad (2)$$

The strains in the slab may be determined from the moments, which are given by the following equations. For a load, P, at mid-span, the moments at any point on the x axis are given by Equation (3).

$$M_x = M_y = \frac{(1+\mu)P}{4\pi} \log_e \cot \frac{\pi x}{2s} \quad (3)$$

M_x and M_y are the bending moments per unit of length, acting on sections normal to the x and y axes, respectively; positive when producing compression at the top of the slab.

The moments at any point on the y axis are given by Equation (4),

$$\left.\begin{matrix} M_x \\ M_y \end{matrix}\right\} = \frac{(1+\mu)P}{4\pi} \log_e \coth \frac{\pi y}{2s} \pm \frac{(1-\mu)Py}{4s \sinh \frac{\pi y}{s}} \quad (4)$$

For a load at a point ($x = v$, $y = 0$) the moments at any point (x, y) are given by Equation (5),

$$\left.\begin{matrix} M_x \\ M_y \end{matrix}\right\} = \frac{(1+\mu)P}{8\pi} \log_e \frac{A}{B} \pm \frac{(1-\mu)Py}{8s} \left(\frac{1}{B} - \frac{1}{A} \right) \sinh \frac{\pi y}{s} \quad (5)$$

wherein $A = \cosh \frac{\pi y}{s} + \cos \frac{\pi (x-v)}{s}$ (6)

and $B = \cosh \frac{\pi y}{s} - \cos \frac{\pi (x+v)}{s}$ (7)

Obviously, for a load at the quarter-point of span, $v = s/4$ in the foregoing equations.

The moments given by Equations (3) to (5) attain infinite values at the load point. Equation (8), based upon the assumption that the applied load is distributed uniformly over a circular area of

diameter c, was developed by Westergaard and applies to the point directly under the load,

$$\left.\begin{matrix} M_x \\ M_y \end{matrix}\right\} = \frac{(1+\mu)P}{4\pi}\left[\log_e\left(\frac{4s}{\pi c_1}\cos\frac{\pi v}{s}\right)+\frac{1}{2}\right] \pm \frac{(1-\mu)P}{8\pi} \qquad (8)$$

wherein $c_1 = 2\,(\sqrt{0.4c^2 + h^2} - 0.675h)$ when $c < 3.45h$

$c_1 = c$ . . . . . when $c \gtrless 3.45h$

h = thickness of the slab.

Values of strain may be computed from the moments indicated above. From the well known relations between strain, stress and moment, it is known that the strain ϵ is given by Equation (9).

$$\epsilon_x = \frac{1}{E}(\sigma_x - \mu\sigma_y) = \frac{c_0}{EI}(M_x - \mu M_y) = \frac{c_0}{N(1-\mu^2)}(M_x - \mu M_y) \qquad (9)$$

wherein σ_x and σ_y = stresses in x and y directions, respectively, tension positive;

ϵ_x = strain in the x direction, elongation positive;

c_0 = distance from neutral axis to point where ϵ is measured, positive downward;

$$N = \frac{EI}{1-\mu^2}$$

Similarly, the strain in the y direction is

$$\epsilon_y = \frac{c_0}{N(1-\mu^2)}(M_y - \mu M_x) \qquad (10)$$

It may be noted that the strains might also be obtained directly from the curvatures of the slab, by means of the relations

$$\epsilon_x = -c_0\frac{\partial^2 w}{\partial x^2} \qquad (11)$$

$$\epsilon_y = -c_0\frac{\partial^2 w}{\partial y^2} \qquad (12)$$

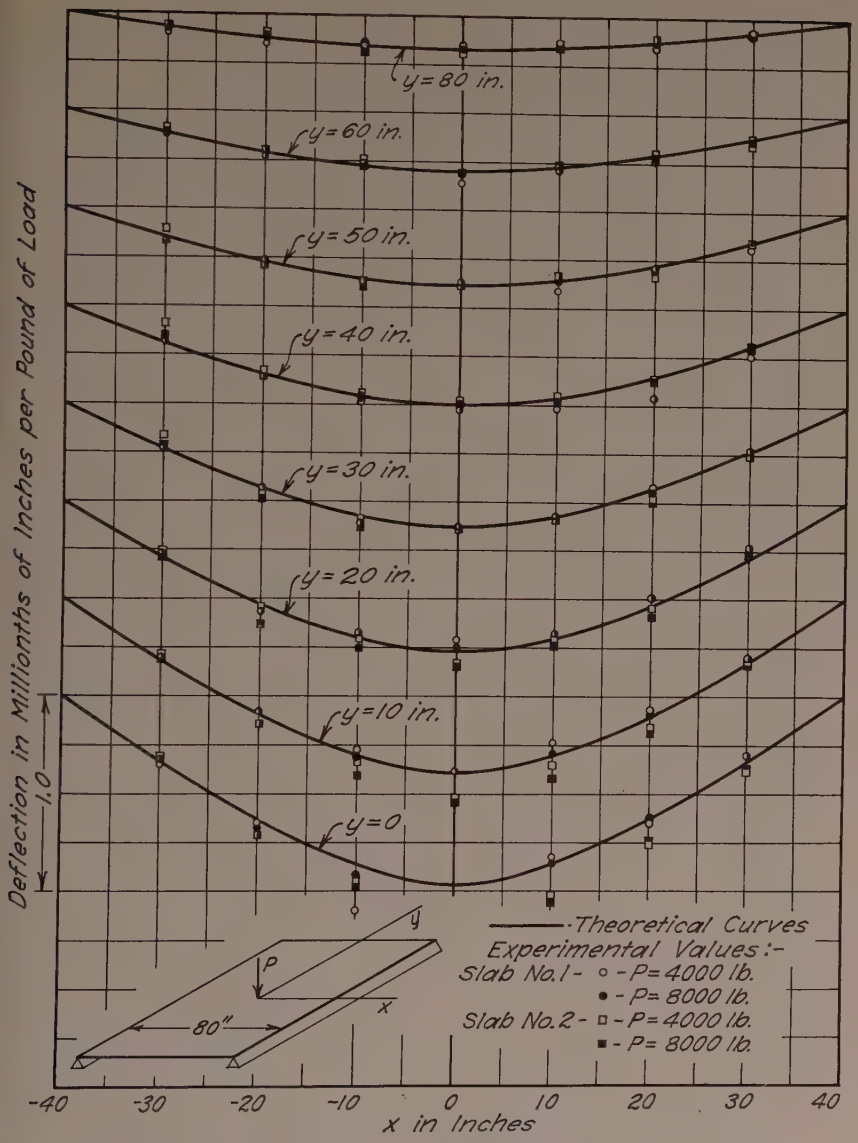


Fig. 11. Deflections of Slab on Sections Parallel to x Axis, with Load at Mid-span

(a) *Deflections.*—Theoretical values of the deflection per pound of load at mid-span and at quarter-point of span for slabs of 80-in. span and infinite width were computed from Equation (1), using a stiffness factor, N = 112 000 000 in.-lb., and are plotted in Figs.

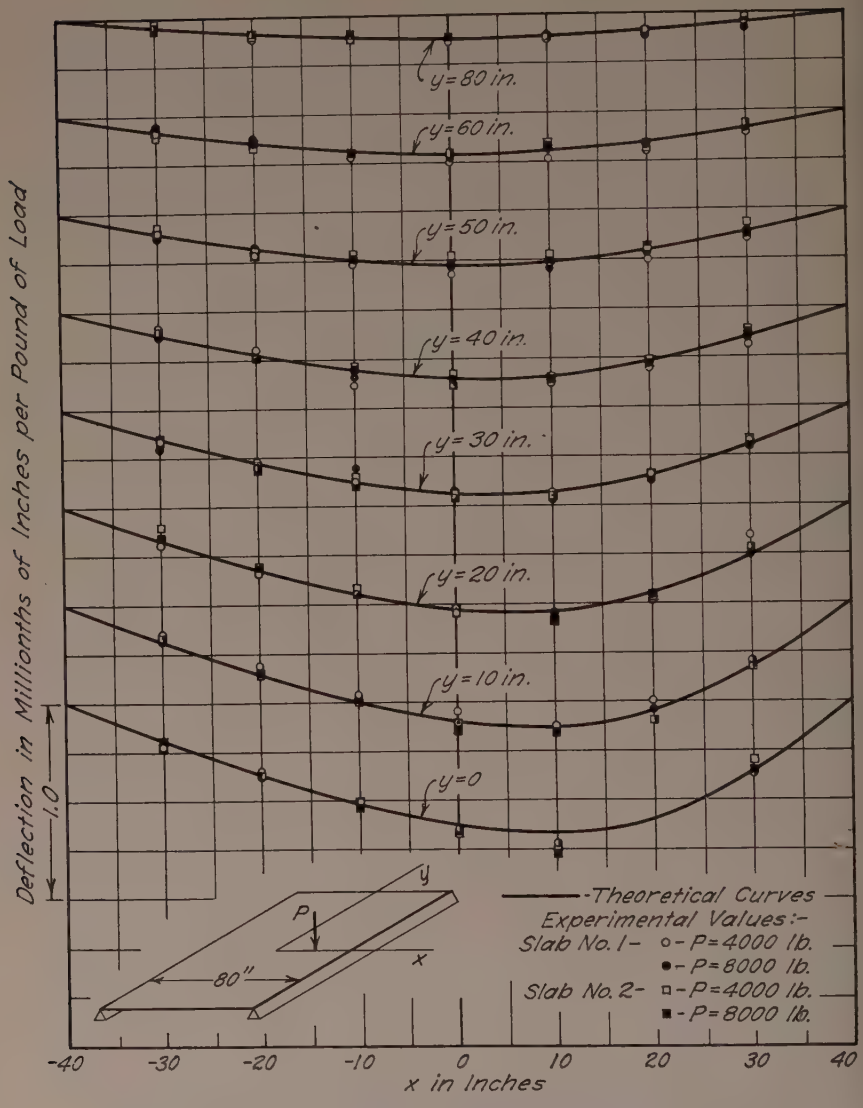


FIG. 12. DEFLECTIONS OF SLAB ON SECTIONS PARALLEL TO X AXIS, WITH LOAD AT QUARTER POINT OF SPAN

11 and 12, respectively. Points representing values of the measured deflections of slabs 1 and 2, in inches per pound of load, are plotted for comparison with the theoretical curves. Since the value of N determined for slab No. 2 was 125 000 000 in.-lb., the measured values for this slab have been multiplied by the ratio $^{125}/_{112}$, to make them comparable with the computed curves and the measured values for slab No. 1. These points were obtained for loads of 4000 and 8000 lb. on both slabs. The fact that these points occupy almost identical positions for the several loads indicates the proportionality between load and deflection of the uncracked slab. The agreement between measured and computed values does not have great weight as a verification of the validity of the assumptions involved in applying the slab theory to these two tests.

(b) *Reactions.*—Theoretical values of the reaction per unit length along one edge of an infinitely long slab having an 80-in. span and resting on unyielding supports were computed and are shown in Fig. 13 for a concentrated load of one pound at distances of ¼, ½, and ¾ of the span from the edge at which the reaction is found. The computations are based on values of Poisson's ratio of 0 and 0.15, though the latter value is undoubtedly preferable. For comparison, points representing the values of the measured reactions per unit load from the tests of slab No. 1 are shown with the theoretical curves. There is a considerable and consistent difference between the computed and the measured values in the vicinity of the load point. This difference is discussed in Appendix A where it is shown that even a slight flexibility of the support has a considerable effect on the distribution of the reaction along the support. It can also be shown that the magnitude of the effect varies with the position of the load. It is estimated that the total compression of the reaction ring, ball bearing, and steel blocks is about 0.002 in. for a load of 1000 pounds on the ring. This flexibility has been assumed in the computations of Appendix A, as well as a condition in which the rings were twice as stiff.

The differences between the measured values of the maximum reaction per unit length at the middle of the support and the values computed for rigid supports, with $\mu = 0.15$, are, in terms of the computed reaction, 10.5 per cent for a load at the three-quarter point, 11.9 per cent for a load at the center, and 32.6 per cent for a load at the one-quarter point, which was nearest the measured reactions. Additional test results indicate that for a load applied directly above a reaction ring, the reaction carried by the loaded ring is approximately 17 per cent of the total load applied.

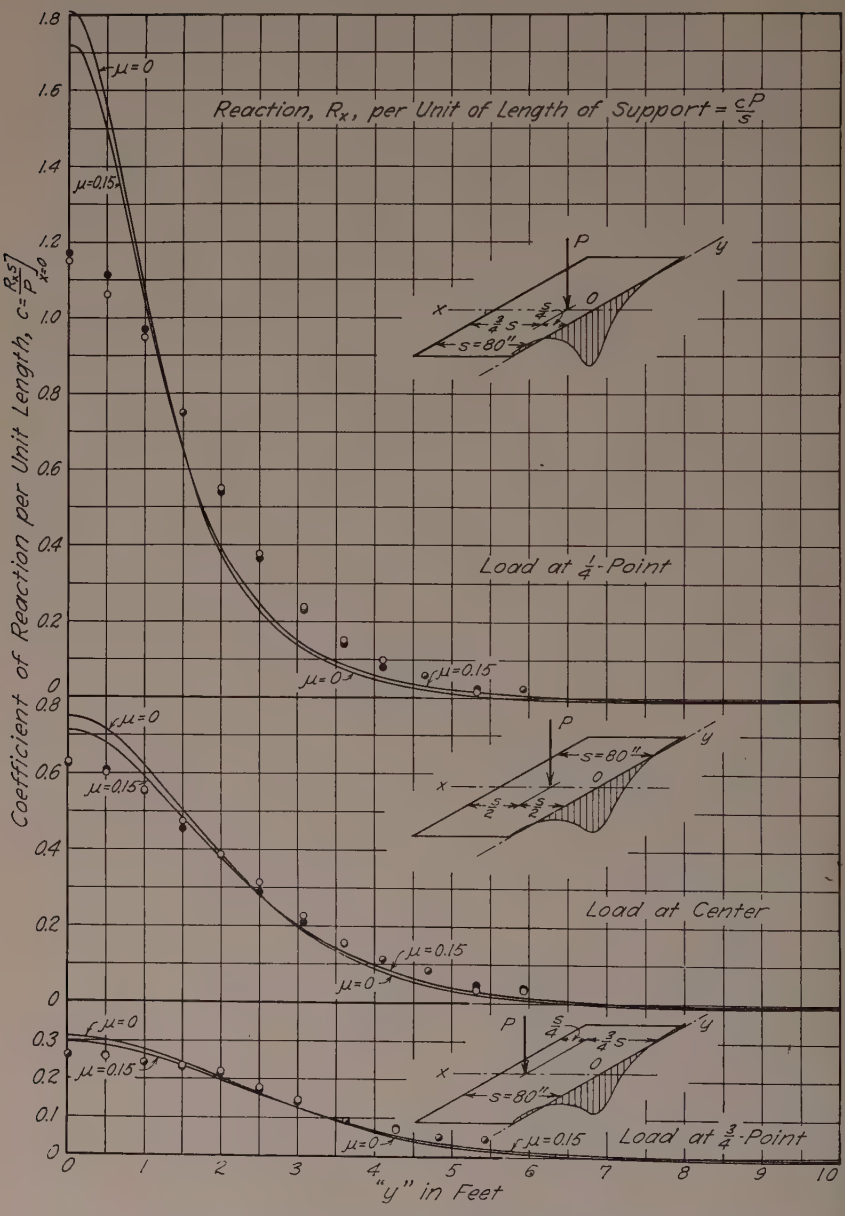


FIG. 13. REACTIONS ALONG ONE EDGE OF SLAB, WITH LOAD AT A DISTANCE OF ¼-, ½-, AND ¾-SPAN FROM THE SUPPORT

A comparison is made of the differences between measured values, computed values for rigid supports, and the computed values of Appendix A for flexible supports in the following tabulation:

Position of Load	Ratio: Measured Reaction / Computed Reaction, Rigid Support	Ratio: Computed Reaction, Flexible Support / Computed Reaction, Rigid Support	
		Rings of stiffness used in tests	Rings twice as stiff as used in tests
Load over support....	0.170	0.189	0.239
Load at mid-span.....	0.881	0.851	0.901

This tabulation indicates that the variation between the measured values and the ordinary computed values of reactions is just about accounted for by the flexibility of the reaction rings. To eliminate the discrepancy between the two values, it would apparently be necessary to maintain the edge of the slab absolutely without deflection. Reaction measuring devices even as stiff as those used in the test are not stiff enough. Even with stiffer devices, minor irregularities in stiffness between adjacent supports and in bearing contact between slab and support will cause an erratic variation of reaction along the edge of the slab.

(c) *Strains.*—Theoretical values of the strains in both the longitudinal and the transverse directions on the center lines of an infinitely long slab having an 80-in. span have been computed for the cases of a concentrated load of one pound at mid-span and at the quarter point of span. These strains were computed from the theoretical values of the slab moments using a value of $\mu = 0.15$ and the value of N determined experimentally for the slab. In order to simplify comparisons it has been found convenient to multiply these strains by the quantity N which represents the stiffness of each individual slab, thus making it possible to represent the weighted strains for the two slabs by a single curve. These values of $N\epsilon$, wherein ϵ is the value of the strain, have been used in plotting Figs. 14 and 15.

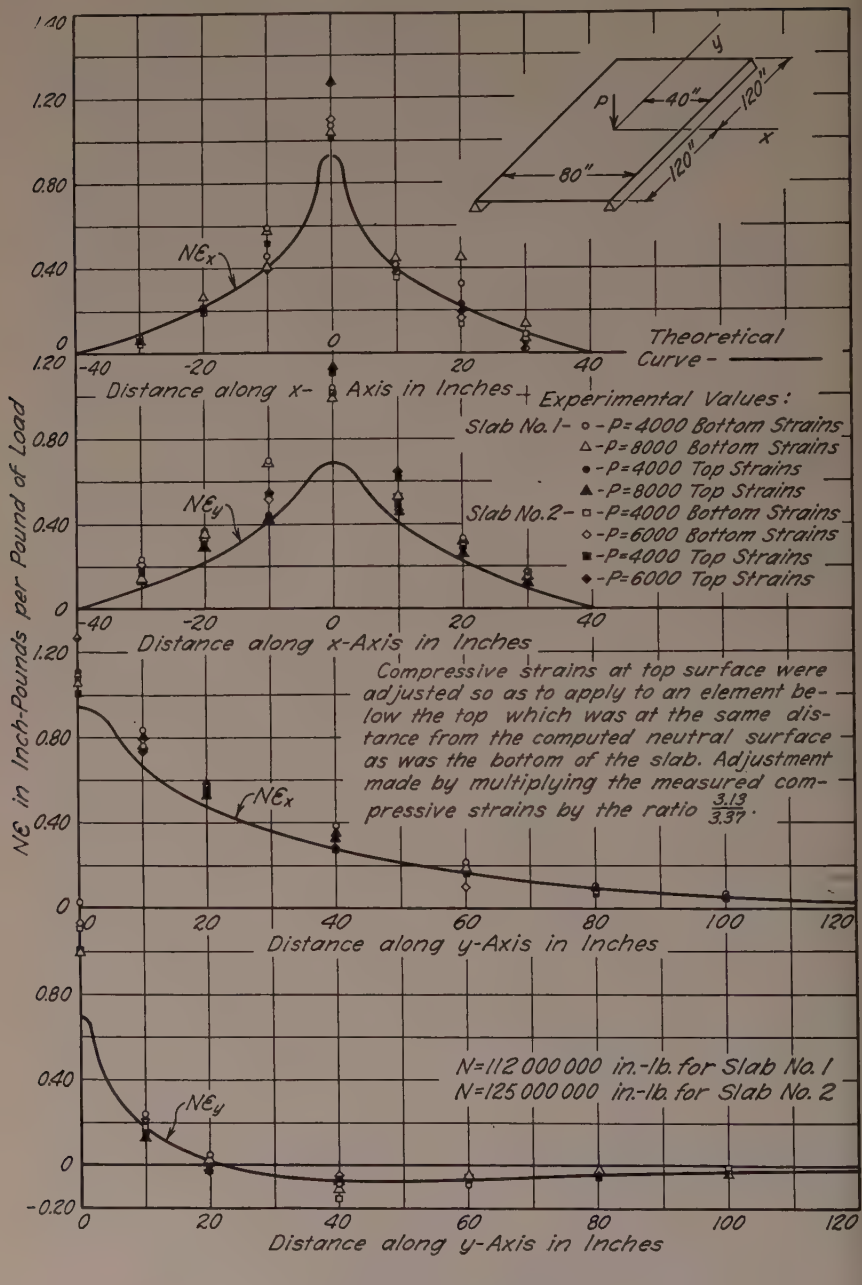

Nε in Inch-Pounds per Pound of Load
Nεx
Nεy
Distance along x-Axis in Inches
Distance along y-Axis in Inches
Theoretical Curve
Experimental Values:
Slab No. 1- ○-P=4000 Bottom Strains
△-P=8000 Bottom Strains
●-P=4000 Top Strains
▲-P=8000 Top Strains
Slab No. 2- □-P=4000 Bottom Strains
◇-P=6000 Bottom Strains
■-P=4000 Top Strains
◆-P=6000 Top Strains
Compressive strains at top surface were adjusted so as to apply to an element below the top which was at the same distance from the computed neutral surface as was the bottom of the slab. Adjustment made by multiplying the measured compressive strains by the ratio 3.13/3.37.
N=112 000 000 in.-lb. for Slab No. 1
N=125 000 000 in.-lb. for Slab No. 2
P
40"
120"
80"
y
x

FIG. 14. VALUES OF WEIGHTED STRAIN, Nε, IN SLAB DUE TO LOAD AT MID-SPAN

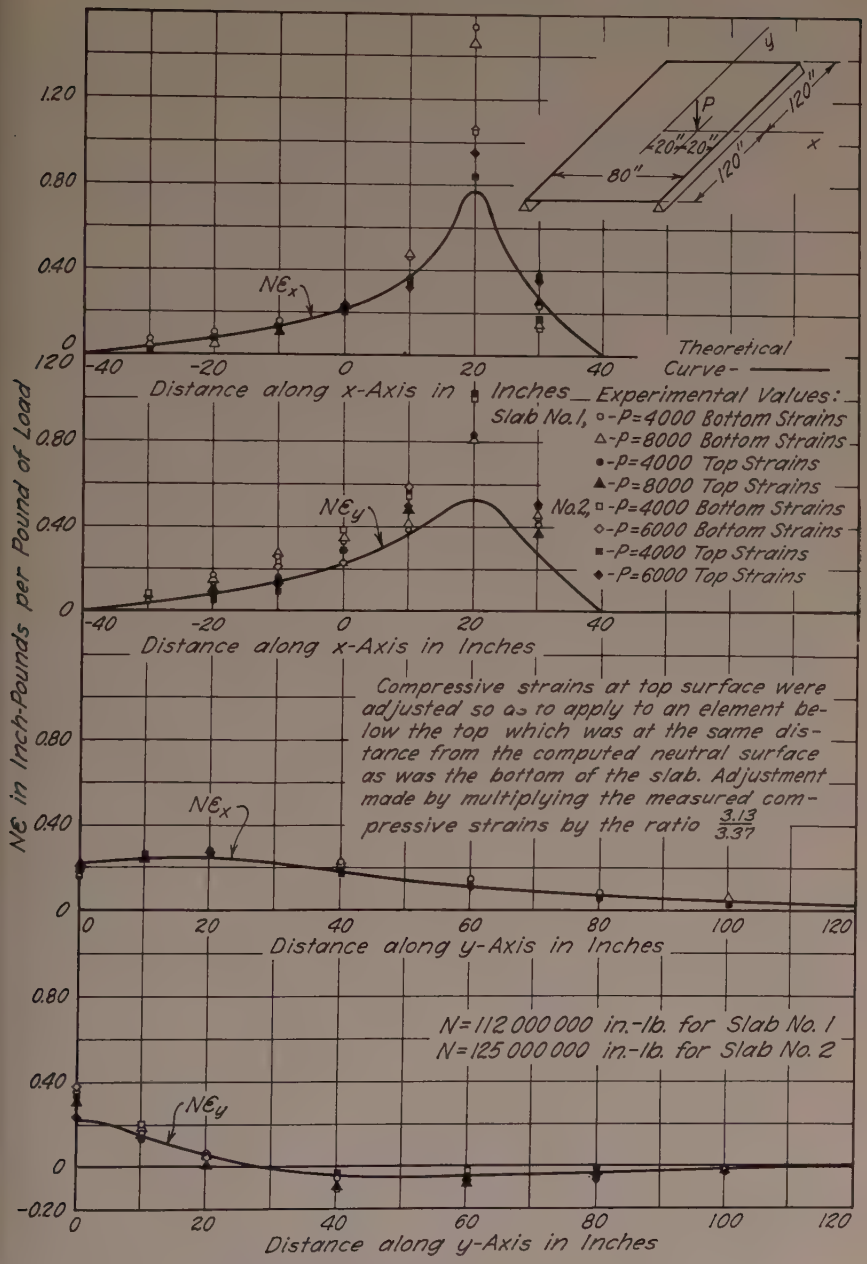

Nε in Inch-Pounds per Pound of Load
Nεx
Nεy
Distance along x-Axis in Inches
Distance along y-Axis in Inches
Theoretical Curve -
Experimental Values:
Slab No.1, ○-P=4000 Bottom Strains
△-P=8000 Bottom Strains
●-P=4000 Top Strains
▲-P=8000 Top Strains
No.2, □-P=4000 Bottom Strains
◇-P=6000 Bottom Strains
■-P=4000 Top Strains
◆-P=6000 Top Strains
Compressive strains at top surface were adjusted so as to apply to an element below the top which was at the same distance from the computed neutral surface as was the bottom of the slab. Adjustment made by multiplying the measured compressive strains by the ratio 3.13/3.37
N=112 000 000 in.-lb. for Slab No. 1
N=125 000 000 in.-lb. for Slab No. 2
P
80"
120"
20"
y
x

FIG. 15. VALUES OF WEIGHTED STRAIN, Nε, IN SLAB DUE TO LOAD AT QUARTER POINT OF SPAN

For comparison, points representing measured value of the strains have also been plotted on Figs. 14 and 15 with the theoretical values. In plotting these points certain adjustments were made so that all points could be shown on a comparable basis. First, consideration was given to the fact that slab No. 1 had an average thickness of 6.56 inches and slab No. 2 had an average thickness of 6.62 inches. However, in both cases, using the experimental value of E, it was found that the neutral surface was located at a distance from the top of the slab equal to 0.519 times the total depth of slab. In plotting the measured points this computed position of the neutral surface was assumed to be the actual one, and the compressive strains were reduced in the ratio of 0.481 to 0.519 to adjust them to the condition that would exist if all strains were measured at an equal distance from the neutral surface. By such adjustments all theoretical and measured strains were reduced to the case of a strain measured at a point 3.13 inches from the neutral surface of the nominal 6½-in. slab.

A study of Figs. 14 and 15 shows a reasonably good agreement between the theoretical and measured strains except for those directly under the load point and for those strains measured on the x axis in the y direction. The points representing measured values of $N\epsilon_x$ along the x axis are generally a little higher (perhaps 10 to 15 per cent) than the calculated values with the load at the center, and much the same with the load at the quarter-point, except that the strains in slab No. 1 were about 90 per cent above the calculated ones under the load point. The measured values of $N\epsilon_y$ on the x axis are pretty generally a considerable amount above the calculated ones, averaging perhaps 40 per cent above the calculated values. This is true both with center loading and load at the quarter-point.

The value of $N\epsilon_x$ on the y axis is fairly close to the calculated value, although the average will show that the measured values are consistently a little above the calculated ones.

The values of $N\epsilon_y$ along the y axis give perhaps the best agreement between measured and calculated values. Both for the load at center and at quarter-point the agreement is very good, except in one case where the strain directly under the load point was approximately 50 per cent above the calculated value.

It is generally recognized that the ordinary slab theory does not apply in regions very close to a concentrated load. The theoretical values of moment at the load point were determined as for a load uniformly distributed over a circle six inches in diameter, using results given by Westergaard, in which a correction is made to the

ordinary slab theory. From results given by Westergaard the following moments are found when the load point is at mid-span and the load is uniformly distributed over a six-inch circular area: $M_x = 0.333P$, $M_y = 0.235P$. For a load concentrated at a point the following moments are found: $M_x = 0.369P$, $M_y = 0.301P$.

The latter values are higher than the former by 10.6 per cent for M_x and 13.3 per cent for M_y. An increase of this magnitude does not explain the very large discrepancy between measured and computed tensile strains under the load point. The discrepancy, together with other evidence, suggests the presence of very fine cracks near the load point. Although the detection of cracks by visual examination at the lower loads is very difficult, fine cracks were observed in both of the slabs at loads of 8000 to 12 000 lb. While these cracks were in many cases so small that a magnifying glass was required for their detection, it seems probable that cracks which would not be discerned visibly would produce a marked effect on the measured strains on the lower side of the slab.

A study of Figs. 14 and 15 shows that with either center loading or with load at the quarter-point, measured tensile strain in the x direction at the quarter-point is greater than is consistent with the other strains. Again the possibility of a very fine crack existing at this point seems to be the best explanation, particularly since a crack was observed at this point at a load of 12 000 lb. in slab No. 1.

The fact that the measured transverse strains ϵ_y on the x axis are consistently higher than the theoretical strains may be partially due to errors in the measured values of N used in the computations. The agreement of measured and computed strains would be better if the values of N used were about 10 per cent smaller. While there was some indication from the test data of the control beams that the value of N should be smaller, it would be hard to determine just how much reduction in N one might justifiably make. It seems sufficient, at present, to point out this possibility, and to conclude that in general the measured strains are in reasonably close agreement with the theoretical ones, both as to magnitude and distribution, except at the point of application of the load, where the measured strains are consistently high.

12. *Concentrated Load Tests Producing Initial Cracks in Slabs.*— This series of loadings, as noted in Section 9, was applied successively to nine points on each slab (points A to I, Fig. 1). The loads were applied gradually, with systematic and frequent observations of strain to aid in the discovery of the first discernible crack at each

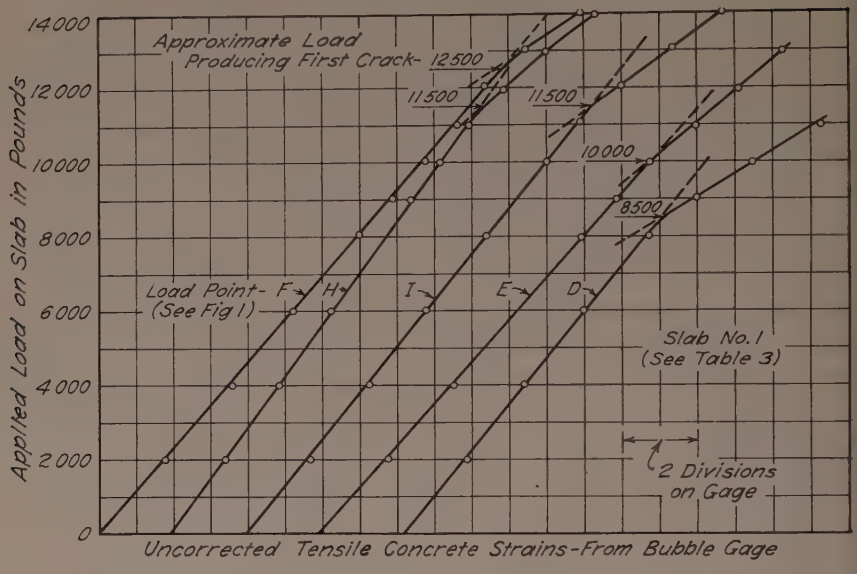


FIG. 16. UNCORRECTED LOAD–STRAIN CURVES USED TO DETECT FORMATION OF CRACKS

of the load points. Strains were measured on the bottom of each slab directly under the load point, in the x direction. In slab 2, after a crack was found in the x direction, the strain gage was mounted in the y direction. Cracks in this direction were generally indicated at a higher load than in the x direction. The surface of the slab was examined closely with the aid of a magnifying glass. In some cases a crack could not be located, even though a change in the direction of the load-strain curve indicated that the crack must exist. Figure 16 shows some of the load-strain curves obtained from the tests. Since these strains, measured with the bubble gage, were measured for the sole purpose of detecting a break in the load-strain curve, they are not corrected for any rotation that may have occurred in the slab. The loads at which cracks were noted visually are shown on these curves and are seen to agree fairly well with the loads at which changes in slope of the curves occur.

A summary of the loads which produced initial cracks in the two slabs is given in Table 3. These loads seem comparatively low; however, it must be remembered that the cracks at these loads were very small, and in general could not be found after the load was removed. The test slabs at this stage may compare well with bridge

TABLE 3
SUMMARY OF LOADS AT WHICH INITIAL CRACKS FORMED IN TEST SLABS

Position of Load		See Fig. 1	Slab No. 1		Slab No. 2	
			Load lb.	Remarks	Load lb.	Remarks
Center line	¼ pt. of span	C	12 000	Slab evidently cracked somewhat below this load, in x direction	10 500	Crack in x direction checked by strain readings
	mid span	A	8 000	Crack not located but indicated by increase in strain readings in x direction		
	¾ pt. of span	B			8 000 10 000	Crack in x direction Crack in y direction Checked by strain readings
20 in. south of center line	¼ pt. of span	F	12 500	Crack 1½ in. long, x direction. No crack at 12 000 lb. applied several times. Strains indicate crack at 12 500	8 500 9 000	Crack in x direction Crack in y direction Checked by strain curves
	mid span	D	8 500	Crack 2 in. long in x direction. Load checked by strain reading	9 000 10 000	
	¾ pt. of span	E	10 000	Strains indicate crack opening in x direction—data uncertain	10 000	Crack in x direction
20 in. north of center line	¼ pt. of span	I	11 500	Crack in x direction indicated by increase in strains at this load	9 500 10 000	Crack in x direction Crack in y direction Checked by strain readings
	mid span	G	13 000	No break in strain readings below load of 13 000 lb. Crack in x direction	10 000 10 500	Crack in x direction Crack in y direction
	¾ pt. of span	H	11 500	Crack in x direction indicated by increase in strains at this load	12 000 12 000	Crack in x direction Crack in y direction Checked by strain readings

slabs in service, which are frequently reported to have no visible cracks after years of service.

While the cracks under discussion were sufficiently large to affect the strains rather markedly, they were very small and existed only

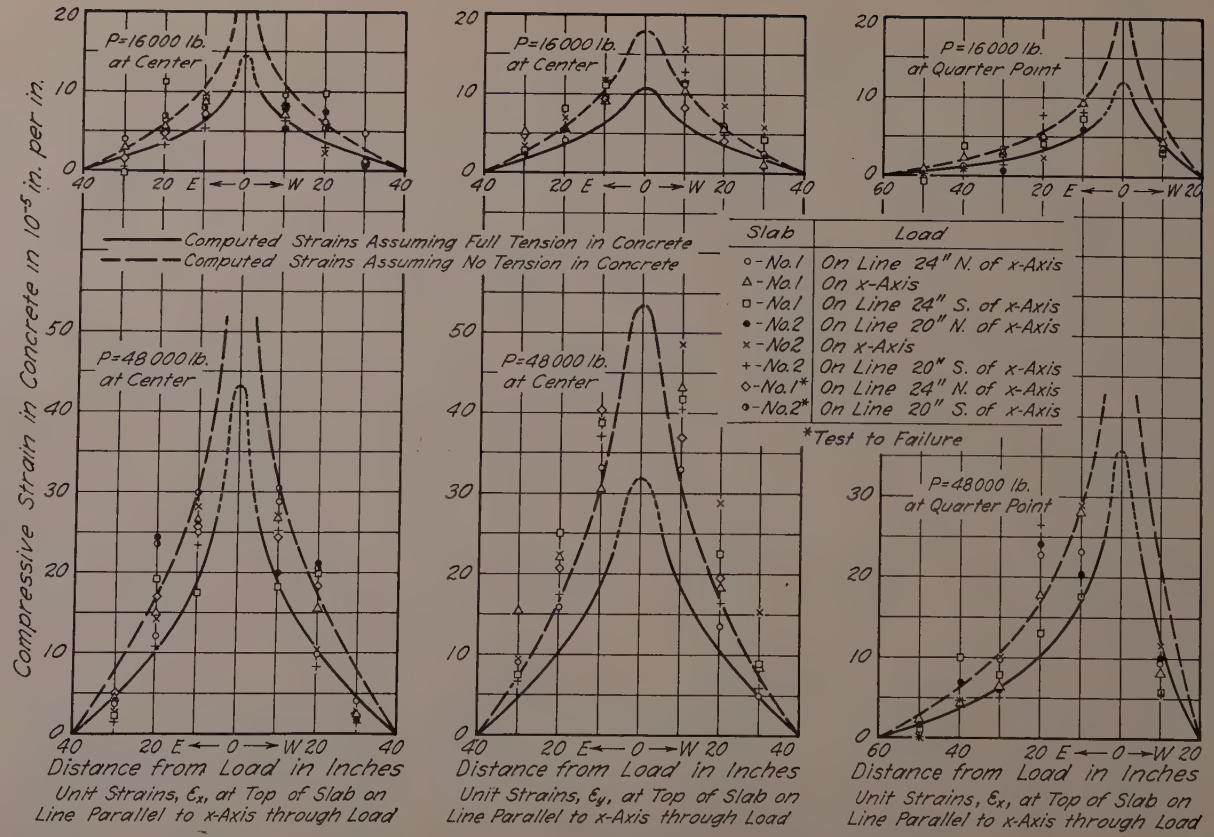

Compressive Strain in Concrete in 10⁻⁶ in. per in.
P=16000 lb. at Center
P=16000 lb. at Center
P=16000 lb. at Quarter Point
P=48000 lb. at Center
P=48000 lb. at Center
P=48000 lb. at Quarter Point
Computed Strains Assuming Full Tension in Concrete
Computed Strains Assuming No Tension in Concrete
Slab | Load
○-No.1 On Line 24" N. of x-Axis
△-No.1 On x-Axis
□-No.1 On Line 24" S. of x-Axis
●-No.2 On Line 20" N. of x-Axis
×-No.2 On x-Axis
+-No.2 On Line 20" S. of x-Axis
◇-No.1* On Line 24" N. of x-Axis
⊙-No.2* On Line 20" S. of x-Axis
*Test to Failure
Distance from Load in Inches
Unit Strains, εx, at Top of Slab on Line Parallel to x-Axis through Load
Distance from Load in Inches
Unit Strains, εy, at Top of Slab on Line Parallel to x-Axis through Load
Distance from Load in Inches
Unit Strains, εx, at Top of Slab on Line Parallel to x-Axis through Load

FIG. 17. VALUES OF CONCRETE STRAINS DUE TO LARGE LOADS

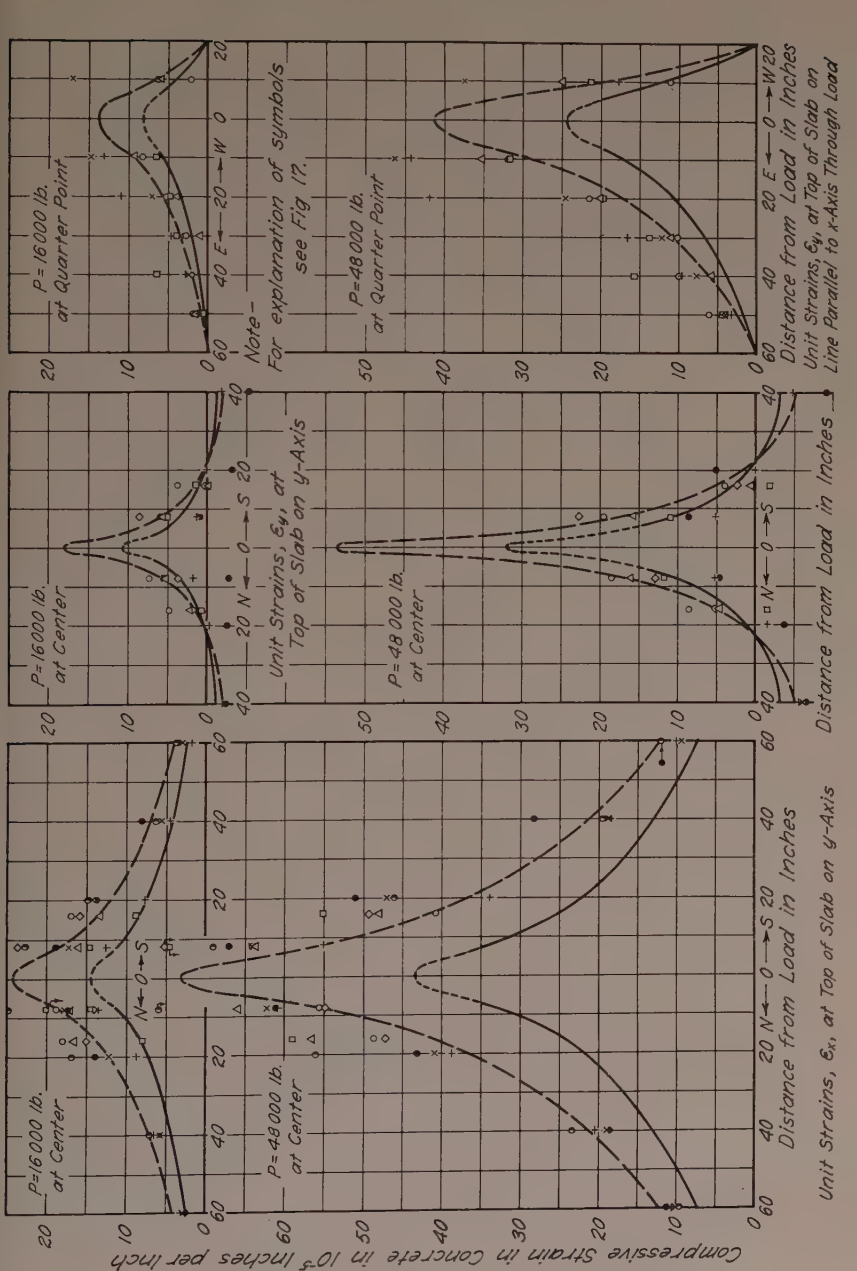

Compressive Strain in Concrete in 10⁻⁵ Inches per Inch
P=16000 lb. at Center
P=48000 lb. at Center
Distance from Load in Inches
Unit Strains, εx, at Top of Slab on y-Axis
Unit Strains, εy, at Top of Slab on y-Axis
P=16000 lb. at Quarter Point
P=48000 lb. at Quarter Point
Note— For explanation of symbols see Fig. 17.
Distance from Load in Inches
Unit Strains, εy, at Top of Slab on Line Parallel to x-Axis Through Load

FIG. 18. VALUES OF CONCRETE STRAINS DUE TO LARGE LOADS

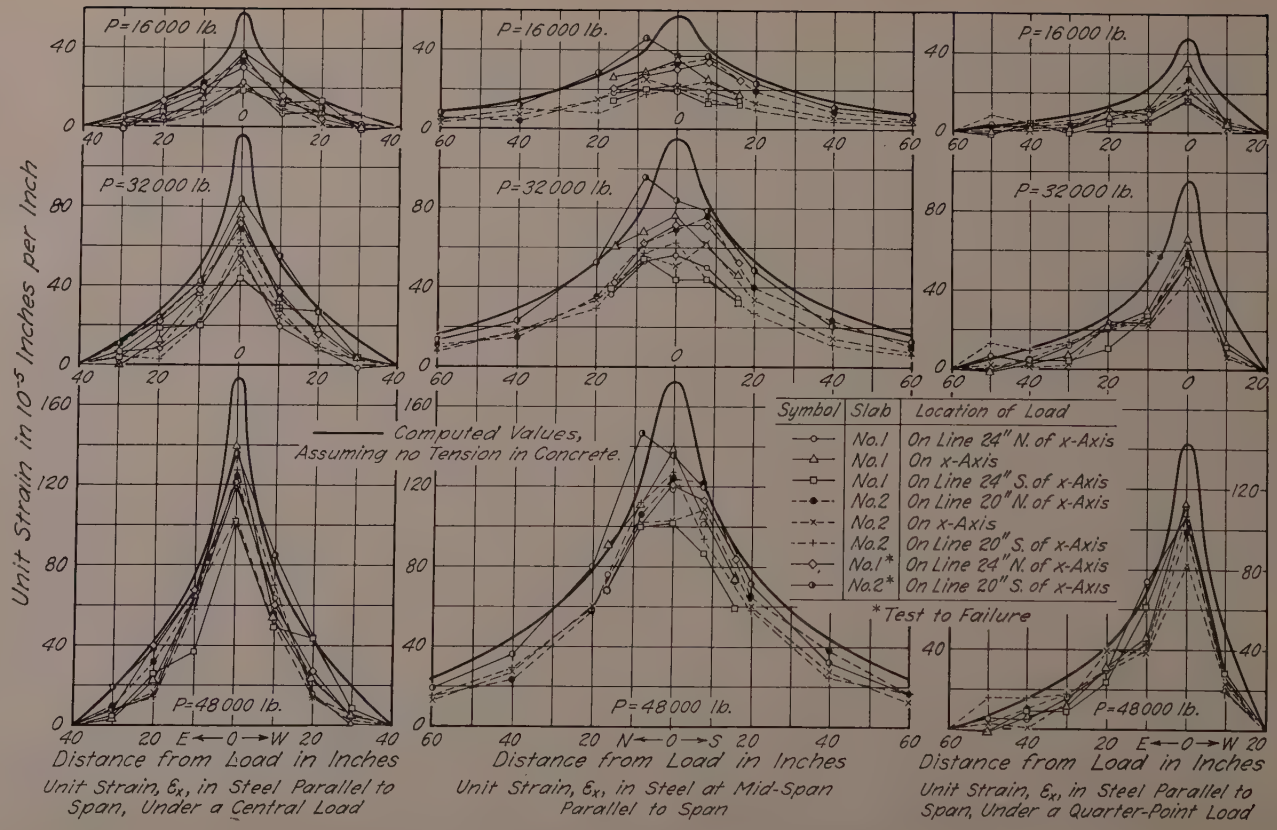

P=16000 lb.
P=32000 lb.
P=48000 lb.
Unit Strain in 10^{-5} Inches per Inch
Computed Values, Assuming no Tension in Concrete.
Symbol | Slab | Location of Load
No.1 | On Line 24" N. of x-Axis
No.1 | On x-Axis
No.1 | On Line 24" S. of x-Axis
No.2 | On Line 20" N. of x-Axis
No.2 | On x-Axis
No.2 | On Line 20" S. of x-Axis
No.1* | On Line 24" N. of x-Axis
No.2* | On Line 20" S. of x-Axis
*Test to Failure
Distance from Load in Inches
Unit Strain, ϵ_x, in Steel Parallel to Span, Under a Central Load
Distance from Load in Inches
Unit Strain, ϵ_x, in Steel at Mid-Span Parallel to Span
Distance from Load in Inches
Unit Strain, ϵ_x, in Steel Parallel to Span, Under a Quarter-Point Load

FIG. 19. VALUES OF STEEL STRAINS DUE TO LARGE LOADS

under the load point; cracking was not at all general until much higher loads were reached. For this reason, these first visible cracks probably are not of great significance, except that they indicate the stage of loading of the test slab beyond which the measured strains and computed elastic strains might be expected to diverge rapidly.

13. *Concentrated Load Tests Producing General Cracking.*—After loads had been applied to the test slabs to produce initial cracking, a series of loads was applied in succession to the nine load points (see Fig. 1). Strain measurements on both the concrete and the tensile reinforcement were taken at loads of 8000, 16 000, 24 000, 32 000, 40 000, 48 000 and 56 000 lb. Values of typical concrete strains measured on the top surface of both slabs No. 1 and No. 2 are shown in Figs. 17 and 18. The steel strains are limited to strains in the bottom layer of steel, which ran in the x direction; samples of these strains are shown in Fig. 19.

Figures 17 and 18 show a wide spread in the values of measured concrete strain, though there is fair agreement between the results from the two slabs. The measured values are considerably greater than the values computed on the assumption of full tension in the concrete, and generally less than the values computed on the assumption of no tension in the concrete. It is obvious that the measured strains should be greater than those computed for the uncracked slab, for not only has the slab developed many cracks at these high loads, but the effective modulus of elasticity at these loads is much less than the initial value used in determining the curves of computed strain. While the strains in the cracked slab were computed by the only method available, an inconsistency is involved, since the slab was considered homogeneous and isotropic for moment determinations, and was assumed cracked when the strains were derived from the moments. With this inconsistency in assumptions, great accuracy cannot be claimed for the computed strains in the cracked slab. The greatest measured strains shown are from 0.00075 to 0.0009, or only $\frac{1}{3}$ to $\frac{1}{2}$ of the value to be expected as an ultimate compressive strain for the concrete.

For comparison with the measured strains in the reinforcement, computed strains have also been plotted in Fig. 19. These strains were computed from moments for a homogeneous slab, for which Poisson's ratio is 0.15. The strains were computed from the moments by use of the conventional relation for reinforced concrete,

$$\epsilon = \frac{f_s}{E_s} = \frac{M}{E_s A_s jd}.$$

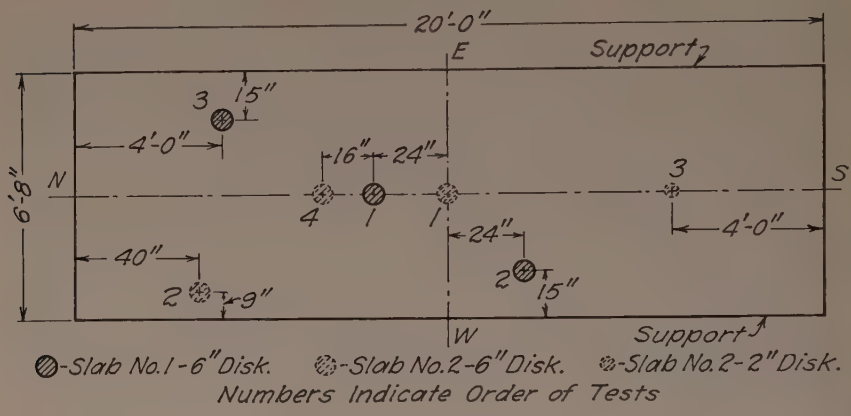


Fig. 20. Location of Points of Application of Loads Producing Local Failure

In this expression, f_s is the steel stress, M is the bending moment, E_s is the modulus of elasticity of the steel, A_s is the steel area and jd is the arm of the resisting couple. Figure 19 shows a very good agreement between measured and computed strains, except directly under the load. Even at this point the measured strains reached about 80 per cent of the computed values at the 48 000-lb. load. The discrepancy may be due to the difficulty of measuring strains at a peak on the moment diagram; it may also be explained by a local bond failure due to the very rapid variation in the stress. Furthermore, it has been noted that the computed values are based on assumptions not strictly rational. However, since this method of computation would probably be used in design, it is of interest that the measured values agree as well as they do with the computed ones.

The greatest steel strains at a load of 48 000 lb. are seen to be approximately 0.0014, corresponding to a steel stress of 42 000 lb. per sq. in. Since the yield point of the reinforcing steel was about 45 000 lb. per sq. in., the greatest stresses at this load were slightly below the yield point stress. This is borne out by the subsequent loadings to produce failure, except in one case, where the yield point was reached at a load of 48 000 lb.

14. *Concentrated Load Tests to Failure.*—After the strain measurements at the 56 000-lb. load, slab No. 1 was tested to failure at three points, sufficiently separated so as not to be affected by the previous local failures of the slab. Load was applied through the 6-in. circular disk by means of a hydraulic jack and the 125 000-lb.

capacity dynamometer. In each case, strain readings were taken in the steel and concrete in the direction of the span until the yield point of the reinforcement had been passed. The load was then steadily increased until final failure occurred.

The position and amount of the three maximum loads carried by the slab were as follows (see Fig. 20):

(1) Load 24 in. north of center of slab; 85 500 lb.

(2) Load 24 in. south of center line and 15 in. east of west support; 77 000 lb.

(3) Load 6 ft. north of center line and 15 in. west of east support; 88 000 lb.

In a similar way slab No. 2 was tested to failure, with loads applied at four different points, as follows:

(1) Load at center of slab, 6-in. disk; 72 000 lb.

(2) Load 40 in. south of north end, 9 in. east of west edge, 6-in. disk; 83 000 lb.

(3) Load 4 ft. from south end, on center line, 2-in. disk; 60 500 lb.

(4) Load 6 ft. 8 in. from north end, on center line; 6-in. disk, 83 700 lb.

From the foregoing, it appears that the location of the load point had little effect upon the value of the maximum load developed. Further, the average load causing failure, using the 6-in. disk, was 83 500 lb. for slab No. 1, and 79 600 lb. for slab No. 2, which shows a very satisfactory agreement in strengths.

Strain measurements indicate that the slab failures did not occur when the yield point of the steel was reached, but required considerably higher loads. In most cases the slab punched through, taking out a section of concrete which was roughly a frustum of a cone, having its smaller base equal to the area of the loading disk and flaring outward very rapidly to meet the plane of the reinforcing steel. The tendency was to strip the reinforcing loose from the remaining portion of the slab. Figure 21 shows a top view of one of the slabs after failure, Fig. 22 a bottom view showing the crack pattern beneath the loaded area, and Fig. 23 another view from below after the reinforcing bars had been cut to permit removal of the cone of concrete that had been punched through.

Figure 24 shows profiles of the fractures of the test slabs, showing quite accurately the shape of the portion punched out. It is evident that the cone or plug of concrete removed was also crossed by many tension cracks, as shown by Fig. 22, so that the outer surface of the cone does not necessarily represent the surface of original failure. It seems likely that the failure due to diagonal tension stresses was a

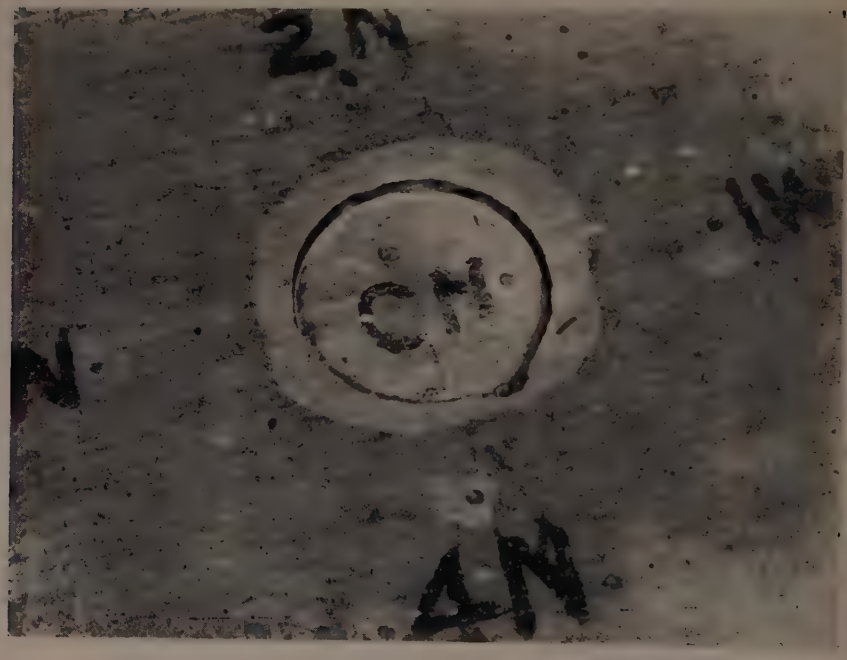


FIG. 21. TOP VIEW OF TEST SLAB AT POINT OF FAILURE

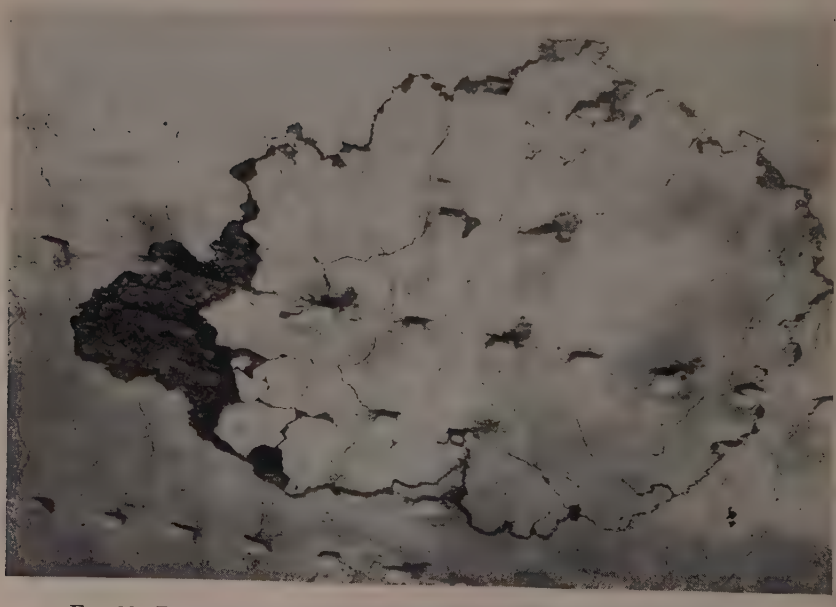


FIG. 22. BOTTOM VIEW OF SLAB BENEATH LOAD POINT AFTER FAILURE

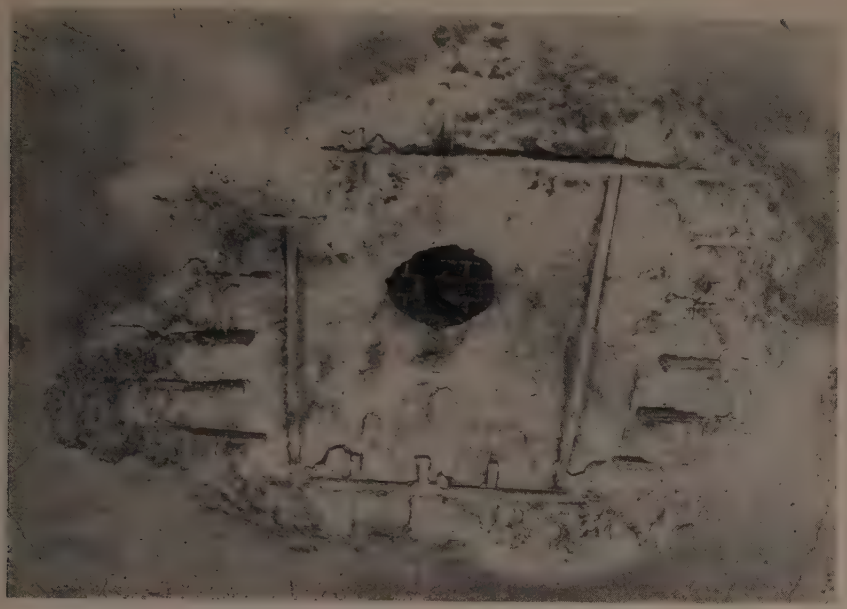
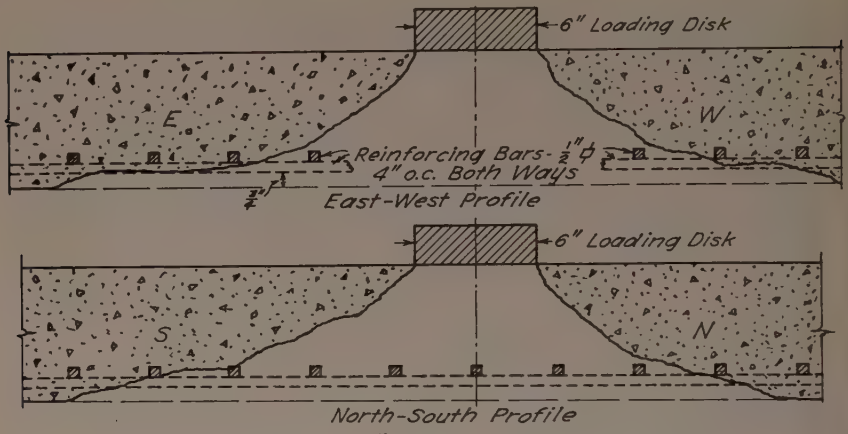


FIG. 23. BOTTOM VIEW OF SLAB SHOWING CONICAL SURFACE OF FRACTURE

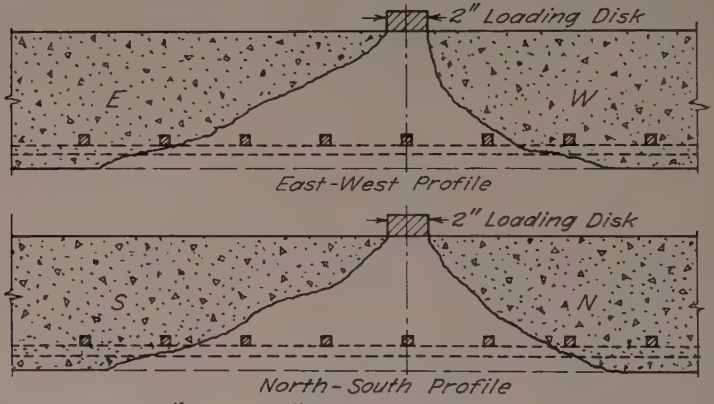
progressive one, with the region of high stress gradually moving upward as cracking progressed. Final failure of the slab in the compression area may have been due to pure shear.

As a study of the resistance of the slab to diagonal tension and shear, it may be noted that the average load carried on a 6-in. disk in six trials on the two slabs was 81 550 lb. and the load carried by a 2-in. disk in one trial was 60 550 lb. By use of a method frequently employed with flat slabs and footings, a design section may be assumed outside the periphery of the loading disk at a distance equal to the effective depth of the slab. Thus with an effective depth of slab equal to 5½ in., a cylindrical section may be found for the 6-in. disk, having a circumference $b = \pi (6 + 11)$ in. and for the 2-in. disk, $b = \pi (2 + 11)$ in. The shearing unit stresses calculated from the equation $v = P/bjd$, are found to be 317 lb. per sq. in. for the 6-in. disk and 306 lb. per sq. in. for the 2-in. disk. The average strength of the concrete, f'_c, in the slabs, as shown by the average value for all of the control cylinders, at all ages, may be taken at 3900 lb. per sq. in. Thus the foregoing values of shearing unit stress may be expressed in terms of the compressive strength of the control cylinders as $v = 0.081\, f'_c$ for the 6-in. disks and $v = 0.079\, f'_c$ for the 2-in. disks. While the selection of the critical section is an arbitrary one, it gives



(a)-6" Disk, 20" South of Middle of Slab

(a)



(b)-2" Disk, 72" South of Middle of Slab

(b)

FIG. 24. PROFILES OF SECTIONS OF FAILURE, SLAB NO. 2

a method of comparing the results of these tests with the diagonal tension resistance of footings, slabs, and beams as found in available test records. In general, it may be said that values of v from tests of beams without web reinforcement may be found which vary from 0.04 to 0.14 f'_c, depending on the amount and anchorage of the longitudinal steel, and on the proportions of the beam.* Tests of

*F. E. Richart, "An Investigation of Web Stresses in Reinforced Concrete Beams," Bul. 166, Eng. Expt. Sta., Univ. of Ill., 1927.

column footings, listed as failing by diagonal tension,* show a range in v/f'_c from 0.05 to 0.13, with an average value of 0.086 (when the value of f'_c for cylinders is estimated from the strength of compression cubes used in the tests). Hence the values found from the present tests appear reasonable and comparable with the other test values quoted.

It is of interest to compare the maximum loads carried by the test slabs with the loads that produced yield point strains in the reinforcing steel. A study of the strain measurements which are available for three of the loadings to failure show that a strain of 0.00152 (which corresponds to the yield point of the steel) was observed directly under the load point at the following loads:

Position of Load	Slab No. 1 Load Point 1	Slab No. 2 Load Point 1	Slab No. 2 Load Point 4
Load producing yield point in steel, lb	57 000	52 000	48 000
Maximum load, lb	85 500	72 000	83 700
Ratio $\frac{\text{Max. load}}{\text{Y. P. load}}$	1.50	1.38	1.74

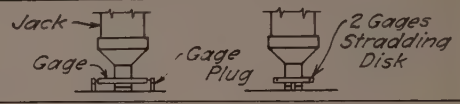
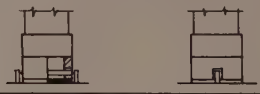
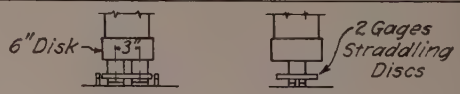
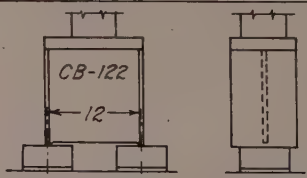
Hence it is seen that the slab carried a considerable increase in load beyond the point at which the yield point was reached in the most highly stressed steel directly under the load point. The increase ranged from 38 to 74 per cent, with an average for the three tests of about 54 per cent. This may be significant in view of the fact that the yield point of the reinforcement is generally considered as the limit of the load-carrying capacity of a reinforced concrete member. It is evident that reaching the yield point in the small area of the slab subject to high stress does not precipitate a general failure of the slab.

III. Tests of Square Slabs to Determine Effect of Size and Shape of Load Bearing Area

15. *Object of Tests.*—As stated in Chapter I of this report, the purpose of the supplementary tests of square slabs was to determine the effect of size and shape of the load bearing area on the variation

*A. N. Talbot, "Reinforced Concrete Wall Footings and Column Footings," Bul. 67, Eng. Expt. Sta., Univ. of Ill., 1913.

TABLE 4
ARRANGEMENT OF LOADS AND STRAIN MEASUREMENTS ON TEST SLABS

Type of Load Area	Size of Load Area	Loading Arrangement and Method of Measuring Strain Under Load on Top of Slab
Circular Disk on Sponge Rubber	2"dia.	
	6"dia.	
	10"dia.	
	14"dia.	
Circular Disk on Plaster of Paris	6"dia.	Same as 6"dia above except on plaster of paris.
Ring on Sponge Rubber	6"O.D. 4"I.D.	
	14"O.D. 12"I.D.	
Pair of Circular Disks on Sponge Rubber	2"dia.	
	6"dia.	

of strain in the concrete and reinforcing steel directly under and in the immediate vicinity of the load. Such tests should have value in the design of concrete slabs in indicating the importance, or the necessity, of considering the size and shape of the loaded area as produced by vehicle tires. It is convenient in the analytical studies

of slabs to apply the load concentration through a circular disk. If the disk is relatively stiff such as the thick 6-in. diameter disk used for the tests reported in Chapter II, there may be a high peripheral pressure caused by the deformation of the slab surface. One of the objects of these tests was, therefore, to study the effect of a peripheral or ring pressure on the strains under the load. Of interest also is the manner of failure as well as the ultimate capacity of the slabs for the various loaded areas.

16. *Outline of Tests.*—These tests were divided into two parts. The tests of series A, on two slabs, each loaded successively through various types of bearing plates, were concerned with the measurement of strain in the concrete on both the top and bottom surfaces of the slab for loads not exceeding 6000 lb. Series B included tests to failure of 18 slabs, 9 of which were companion tests, in which strains were measured in the reinforcing steel only at the bottom of each slab.

Strains were measured over a $6\frac{1}{2}$-in. gage length at the center of the slab, directly under the load, and at two points equally spaced on either side of the center in the direction of and along the longitudinal center line.

The slabs were simply supported along two edges, and the loads applied to the slab through disks and rings of various diameters in the manner indicated in the loading schedule of Table 4.

17. *Description of Test Slabs.*—The slabs were 5 ft. 4 in. by 5 ft. 0 in. and 6 in. thick, reinforced with $\frac{1}{2}$-in. square deformed bars, having the following physical properties, as found from tests of several samples:

Tensile yield point, 44 300 lb. per sq. in.
Ultimate tensile strength, 74 000 lb. per sq. in.

The bars were spaced 5 in. apart in the direction of the span, 10 in. apart in the transverse direction, and all were anchored at the ends with hooks of 4-in. diameter. Each slab had an effective depth of 5 in. and a span from center to center of supports of 5 ft.-0 in. thus allowing a 2-in. overhang beyond the supports. The slab rested on cold-rolled steel bearing plates, 4 in. wide, $1\frac{1}{4}$ in. thick and 5 ft. long, grouted to the slab with neat cement mortar to insure uniform bearing. The plates in turn were supported on the heads of railroad rails embedded in concrete piers.

The proportions of cement, sand, and gravel in the concrete used for the slabs were $1 : 3\frac{3}{4} : 4\frac{1}{4}$, by weight, corresponding to

FIG. 25. VIEW SHOWING METHOD OF LOADING TEST SLABS, SERIES B

1 : 3¼ : 3½, by volume. A water-cement ratio of 1.0 was used. This mix made a fairly stiff concrete, averaging about a 3-in. slump, and was vibrated into place. The control cylinders developed an average strength of 3800 lb. per sq. in. at 28 days, tested dry.

The slabs were cured in the forms with the top surface covered with wet burlap for a period of one week, and then were removed and allowed to dry out in the laboratory air for an additional period of 21 days before testing. All of the control cylinders were cured in the same manner as the slabs.

18. *Description of Tests.*—Load was applied to the slab through disks of various sizes by means of a jack bearing against the head of a testing machine, as shown in Fig. 25, and was measured with a 20 000-lb. dynamometer in the tests of series A and a 125 000-lb. load measuring ring in the tests of series B (both described in Section 6, Chapter II). The loading disks and rings were cut from structural steel 2¾ in. thick, each having its bearing surfaces carefully machined. The disks had diameters of 2, 6, 10 and 14 in., and the rings had outside diameters of 6 and 14 in. with 4- and 12-in.

inside diameters, respectively. To insure uniform pressure over the surface of a large disk, smaller disks were pyramided between it and the jack above. Load was distributed to the tandem 6-in. disks through a short length of 12-in. wide-flange I beam standing on end. The ends of the beam section were milled and the web coped slightly at one end so that only the edge of the flanges were bearing along the diameter of each disk, a bearing plate $1\frac{1}{2}$ inches thick being placed between the other end and the jack. The rings were loaded through disks of the same diameter. The method of applying load to the various disks is shown in the diagrams of Table 4 and in Fig. 25.

Strains in the concrete were measured with the graphic strain gage, a type of fixed instrument (described in Section 8, Chapter II) having a nominal gage length of $6\frac{1}{2}$ in. The gages were actuated by $\frac{5}{16}$-in. hexagonal brass plugs inserted with plaster of Paris in small holes provided along the longitudinal center line of the slab. In order to measure strain at the center for the 2-in. diameter disk, a pair of gages were used to straddle the disks, both gages actuated by a common bracket fastened to gage plugs as shown in Table 4. The plugs were spaced $6\frac{1}{2}$ in. apart so that each one except the end plugs served for two gages. Grooves, 1 in. by $1\frac{1}{4}$ in. milled in the bearing surface of the 6-, 10-, and 14-in. disks, as well as the 6-in. and 14-in. ring, provided space for the graphic gages on the top surface of the slab directly under the load. Steel strains were measured with a Berry strain gage, also having a $6\frac{1}{2}$-in. gage length, at gage lines corresponding to those in the concrete.

The tests of series A started with the larger bearing areas and a maximum load of 6000 lb. each. As the loaded area was decreased the maximum applied load was likewise decreased, although it was divided into at least three increments and strains measured for each. The graphic gages require that the record slide be shifted to define the strain for each increment of load, therefore it was necessary to release the load after each increment and remove the disks to adjust those gages operating underneath the disk.

This procedure was satisfactory for the tests of series A, but proved to be quite unsatisfactory for tests of series B, because it destroyed the continuity of a test which was affected by the various stages of cracking in the slab. Several devices for adjusting the gages under the disk were tried but were unsuccessful, therefore the measurement of concrete strains was omitted in series B, and steel strains only were measured along the center longitudinal reinforcing bar. The slabs were loaded in increments of 2000 lb., and strain readings taken until the reinforcing steel was stressed to the yield point.

From this stage of the test to failure load was applied slowly and uniformly with the testing machine.

19. *Results of Tests of Series A.*—The data obtained from the two tests of series A were reduced to unit strain per pound of load and the values corresponding to each of the three increments of load averaged. Inasmuch as the load-strain relation was practically linear for the relatively small strains measured, such an average was permissible. The concrete strains as measured and here reported are slightly larger than at the slab surface, due to the fact that the gages were operating at a level $\frac{5}{16}$ in. above the slab. No adjustment of the strains to the slab surface was made because the values were for comparative purposes only.

The variation of strain in the concrete along the longitudinal center line on both the top and bottom of the slab for the various load bearing areas is shown in Fig. 26. A comparison for the 2-, 6-, 10- and 14-in. disks is given in Fig. 26. From a comparison of the results for the 6- and 14-in. disks, it is found that the difference between the smallest and the largest strains at the center amounts to 25 per cent of the maximum on top and 14 per cent at the bottom of the slab. The low values indicated for the 2-in. disk are apparently due to the measurement of strain over a relatively long gage length. This effect of size of disk will also be noted in a later figure. It is interesting to note that the maximum strains at the middle of the slab were consistently higher on the bottom than on the top of the slab, although the measured strains on either side of the middle were of nearly the same magnitude as the corresponding values on top (see Fig. 26). Theoretically the strain due to flexure of an uncracked slab reinforced as these were should be slightly greater on the top than on the bottom surface.

Also shown in Fig. 26 are the strains due to applying a load through a 6-in. ring, a 6-in. disk bearing on sponge rubber and a 6-in. disk bearing on plaster of Paris. It appears from these curves that the type of bearing, whether sponge rubber or plaster of Paris, makes practically no difference in the strain distribution. At the bottom of the slab the ring produced a slightly higher maximum strain than the disks, but on the top there appears to be no essential difference.

The 14-in. disk, on the other hand, produced a maximum strain at the bottom of the slab about 14 per cent greater than the 14-in. ring, although the strains on the top due to the ring and the disk do not differ materially. The comparison is shown in Fig. 26. The

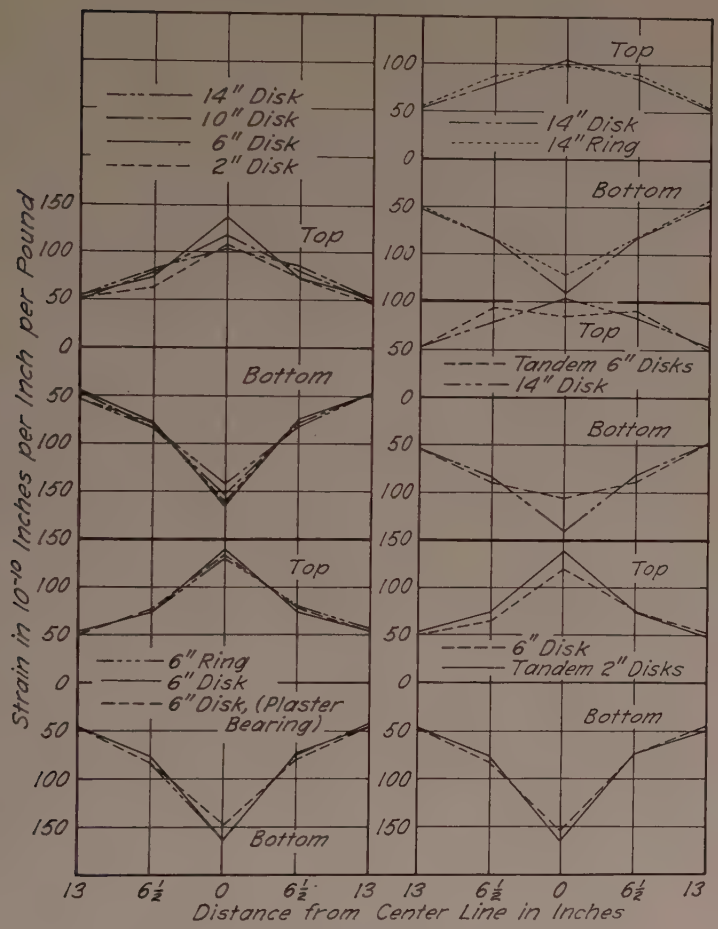


FIG. 26. VARIATION IN STRAIN IN CONCRETE ALONG LONGITUDINAL CENTER LINE

strains due to loading two 6-in. disks and two 2-in. disks placed in tandem are compared with those of the 14-in. disk and 6-in. disk, respectively, in the same figure. The 6-in. disks arranged in this manner definitely modified the distribution of strain, the maximum values occurring under each disk on the top surface. At the bottom the maximum strain occurred at the center of the slab, and is about 25 per cent less than that due to the 14-in. disk. The tandem 2-in. disks produce about the same general distribution of strain as the single 6-in. disk, although the maximum value on the top is 14 per

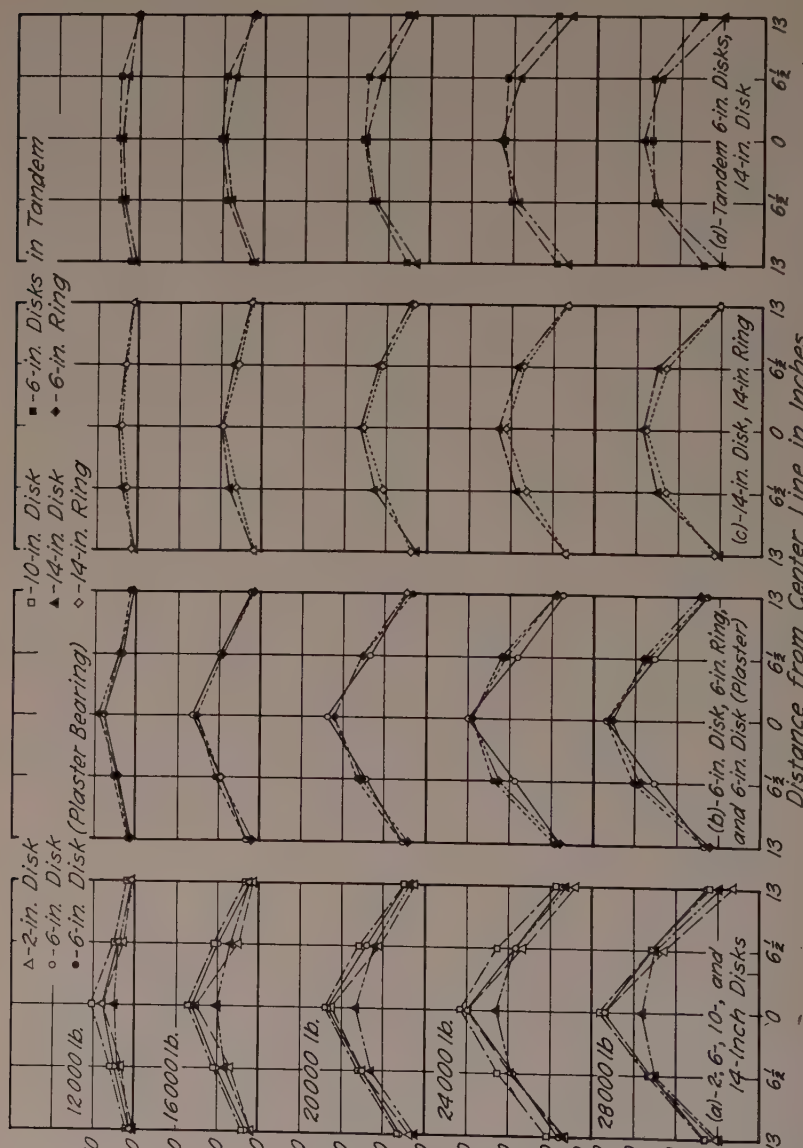


FIG. 27. VARIATION OF STRAIN ALONG CENTER REINFORCING BAR

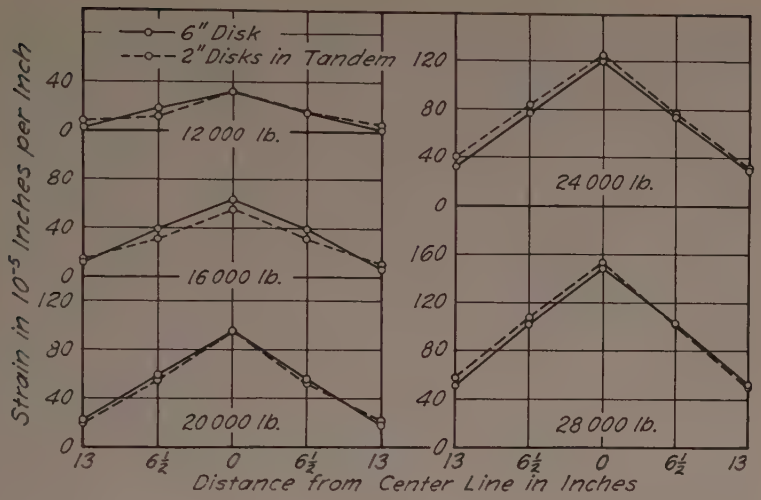


FIG. 28. VARIATION OF STRAIN ALONG CENTER REINFORCING BAR

cent less and on the bottom 6 per cent less than the corresponding maximum values due to the 6-in. disk.

20. *Results of Tests of Series B.*—The variation of strain along the center reinforcing bar as produced by the various load bearing areas is shown in Figs. 27 and 28, for several applied loads. In each case the values plotted represent the average obtained from duplicate tests. In Fig. 27(a) the comparison of strains produced by disks ranging in size from 2 in. to 14 in. in diameter is shown. There is a very definite reduction in strain at the center of the slab due to the 14-in. disk; it amounts to as much as 50 per cent of the largest strain at the 12 000-lb. load. It is peculiar that the 10-in. disk consistently produced the largest strains. Although it is quite possible that the gage was too long to register the effect of a peak strain due to the 2-in. disk, it seems improbable that such was the case for the 6-in. disk. There was close agreement between strain data obtained from duplicate tests of each type of loading area, particularly at the center of the slab, so that the values plotted represent fairly accurately the strain as measured at the various points along the reinforcing bar.

The comparison of the strain curves of Fig. 27(b) for the 6-in. disk bearing on rubber or plaster of Paris and the 6-in. ring indicates no essential difference between them at any load. The same is true of the comparison between the 14-in. disk and 14-in. ring shown in Fig. 27(c). The tandem 6-in. disks appear to produce a slightly

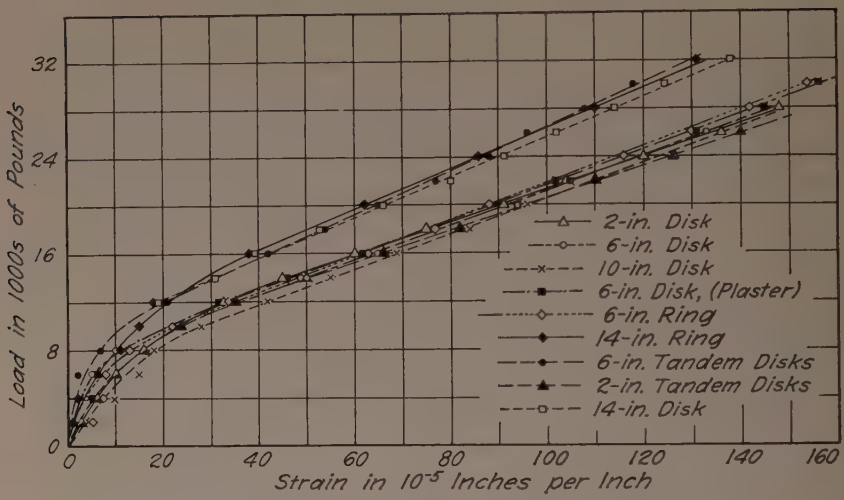


FIG. 29. LOAD–STRAIN CURVES FOR REINFORCING BAR DIRECTLY UNDER LOAD

different strain along the bar from that due to the single 14-in. disk, the difference being more marked at the higher loads. This comparison is shown in Fig. 27(d). Little difference is seen in Fig. 28 between the curves for the 6-in. disk and those for the tandem 2-in. disks.

Load-strain curves for the reinforcing bar directly under the load are shown for the various load bearing areas in Fig. 29. The curves are definitely divided into two groups, those corresponding to the 14-in. disk, the 14-in. ring, and the tandem 6-in. disks comprising one group (having the smaller strains for a given load), and the second group of curves corresponding to disks and ring less than 14 in. in diameter.

Final failure of all the slabs was due to punching through of the disks or rings in the manner shown in Fig. 30, although in every case except those loaded with the 2-in. disk the reinforcing steel had passed the yield point at a load of 60 to 65 per cent of the ultimate. The slabs tested with the 14-in. disk and 14-in. ring exhibited a slight compression spalling of the concrete along the center line on each side of the loaded area at about the same time as punching occurred, indicating a tendency for the slab to fail as a beam when these bearing plates were used. The effects of diameter of disk on the ultimate load, on the load at which yield point stress occurred in the steel, and on the strains at a given load are shown in Fig. 31.

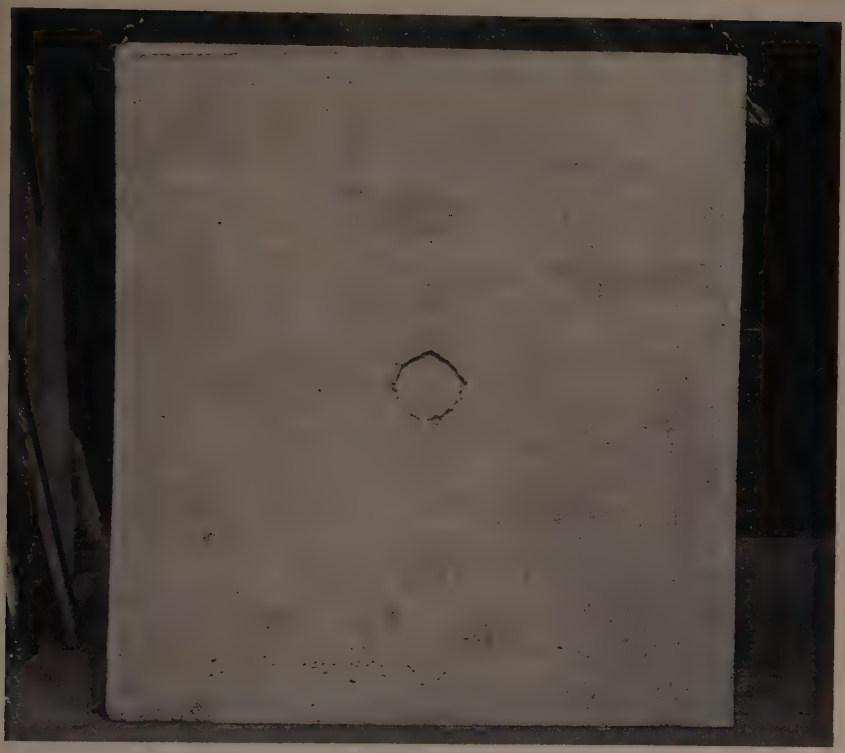


FIG. 30. VIEW OF TEST SLAB AFTER FAILURE

In Fig. 31(b), in addition to the test values, calculated values of the load required to produce a yield-point stress of 44 300 lb. per sq. in. in the steel are shown. These were computed from the theoretical moments based on a value of Poisson's ratio of 0.15, and using the conventional equation for steel stress, $f_s = M/A_s jd$, as in Section 13. Two features of the computed curve are noteworthy: it shows an upward trend with increase in disk diameter, and it shows a very fair agreement with the curve for measured values, despite the fact that the measured strains were taken directly under the load point.

The results of all the tests of series A and B are summarized in Table 5. There is consistently close agreement between the load data for companion tests of series B, but the same cannot be said of the strain data of series A. Many of the tests were repeated; the repeat values for each slab, however, differed little from the original

TABLE 5
SUMMARY OF TESTS TO DETERMINE EFFECT OF LOADING AREA

Loading Area		*Strains in Concrete Under Load 10^{-10} in. per in. per lb.		†Load at Yield Point Stress of Steel lb. per sq. in.	†Ultimate Load lb. per sq. in.	Ratio of Load at Yield Point of Steel to Ultimate Load	Shearing Unit Stress v lb. per sq. in.
		Top	Bottom				
2-in. disk	1	111	173	27 200	34 000	0.80	206
	2	108	149	28 800	37 400	0.77	226
	Av.	110	161	28 000	35 700	0.78	216
6-in. disk	1	132	175	28 600	45 200	0.63	206
	2	145	153	28 000	44 500	0.63	202
	Av.	139	164	28 300	44 850	0.63	204
6-in. (plaster)	1	125	165	28 600	45 000	0.64	205
	2	140	132	28 400	44 800	0.63	203
	Av.	133	149	28 500	44 900	0.64	204
10-in. disk	1	111	162	27 600	49 400	0.56	180
	2	127	143	27 400	48 500	0.57	176
	Av.	119	153	27 500	48 950	0.56	178
14-in. disk	1	101	148	33 600	54 600	0.62	166
	2	106	133	34 400	54 700	0.62	166
	Av.	104	141	34 000	54 650	0.62	166
14-in. ring	1	97	127	37 200	56 000	0.66	169
	2	98	117	32 400	53 700	0.60	163
	Av.	98	122	34 800	54 850	0.63	166
6-in. ring	1	136	173	28 600	45 600	0.63	207
	2	144	153	29 600	45 500	0.65	207
	Av.	140	163	29 100	45 550	0.64	207
2-6-in. disks in tandem	1	92-96	99-105-96	36 400	56 700	0.64	175
	2	96-88	80-105-79	34 800	55 400	0.63	171
	Av.	94-92	90-105-88	35 600	56 050	0.64	173
2-2-in. disks in tandem	1	118	169	27 200	42 300	0.64	221
	2	119	141	26 400	40 000	0.66	209
	Av.	119	155	26 800	41 150	0.65	215

*Average of 3 increments of load under 6000 lb., tests of series A, on two slabs, only.
†From tests of series B, on a total of eighteen slabs.

values, so that the discrepancies are in all probability due to individual differences in the slabs.

21. *Effect of Type of Reinforcement on Manner of Final Failure.*— The results of a series of ¼-scale model tests corresponding to the tests of series B indicated that the manner of final failure might be influenced by the surface condition of the reinforcement or by some other property of the steel such as its ductility. Two additional full scale slabs were therefore constructed and tested, one with ⅝-in.

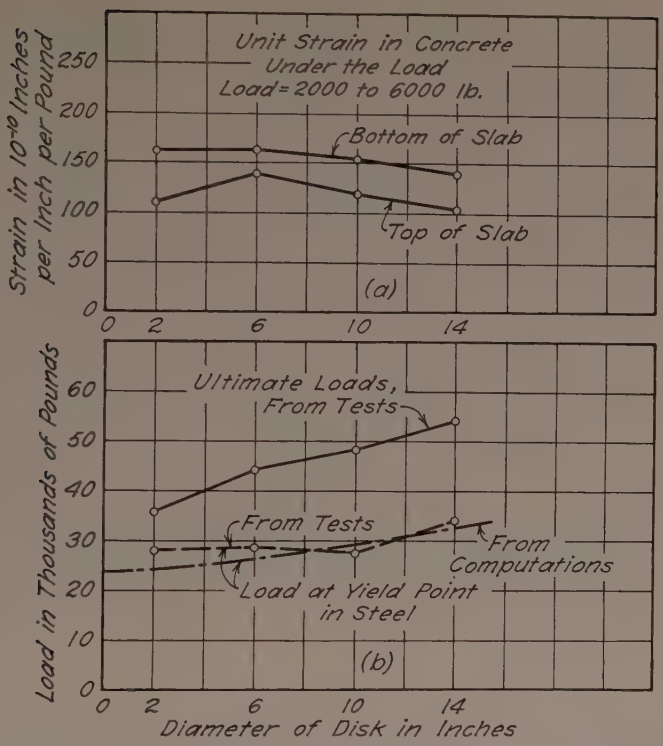


FIG. 31. RELATIONS BETWEEN DIAMETER OF LOADING DISK AND YIELD POINT AND ULTIMATE LOADS

plain round bars having physical properties similar to the ½-in. square bars used in series A and B, and spaced so as to provide the same percentage of steel as in those tests; the other slab containing ½-in. square deformed bars previously annealed for the purpose of increasing their ductility. However, annealing did not increase the ductility as much as anticipated, and did reduce the strength considerably. The following is a comparison of the bar strengths:

Kind of Bar	Yield Point Stress lb. per sq. in.	Ultimate Strength lb. per sq. in.
½-inch square deformed	44 300	74 000
⅝-inch plain round	43 100	73 800
½-inch square deformed (annealed)	36 800	68 500

Both slabs exhibited a flexural failure similar to that of a wide beam, as shown in Fig. 32. The ultimate loads were 42 400 lb. for the slab made with $\frac{5}{8}$-in. bars, and 42 800 lb. for the slab made with $\frac{1}{2}$-in. bars. These values are only slightly less than the corresponding slab strengths of series B, although the concrete strengths in the two cases were about the same. A comparison of the crack patterns of the slabs clearly shows the influence of the deformed bar in resisting bond slip near the final stages of failure, thereby distributing the cracks and bringing into action a greater portion of the slab. Had the annealed $\frac{1}{2}$-in. bar been as strong as the $\frac{5}{8}$-in. bar the difference in ultimate loads would probably have been more marked.

The reduced yield point stress of the annealed bars rather than any increased ductility undoubtedly caused the slab containing these bars to fail as it did. The slab containing the $\frac{5}{8}$-in. plain round bars, on the other hand, evidently failed by the opening of a single large crack because of bar slip at loads near failure. Apparently the concrete reached its ultimate strength in flexure before the diagonal tensile stresses became large enough to produce a punching failure.

22. *Comparison of Ultimate Loads.*—Table 5 shows a variation in the load producing yield point stress in the reinforcement from 26 800 to 35 000 lb., and in the ultimate load from 35 700 to 56 050 lb. Since all of these slabs exhibited a diagonal tension or punching type of final failure, the ultimate load might be expected to depend upon the shearing unit stress. Values of the shearing unit stress, v, in Table 5, have been computed from the formula

$$v = \frac{P}{bjd}$$

wherein

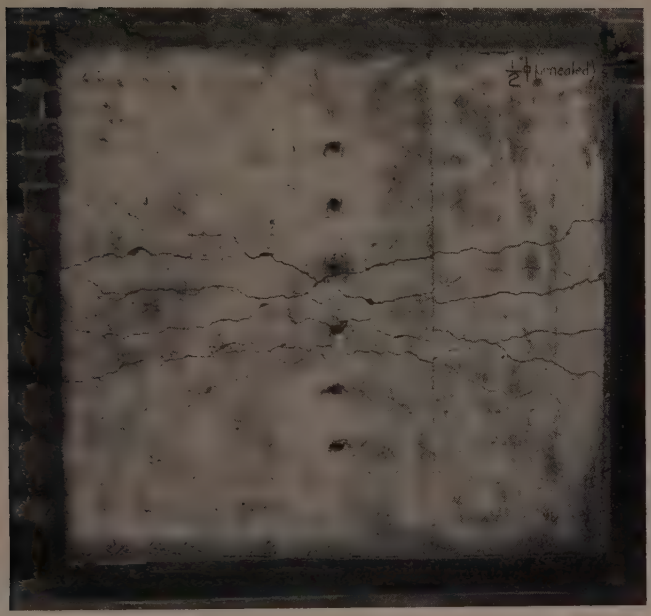
P = ultimate load, applied to the loading disk of diameter D

d = the effective depth of the test slab

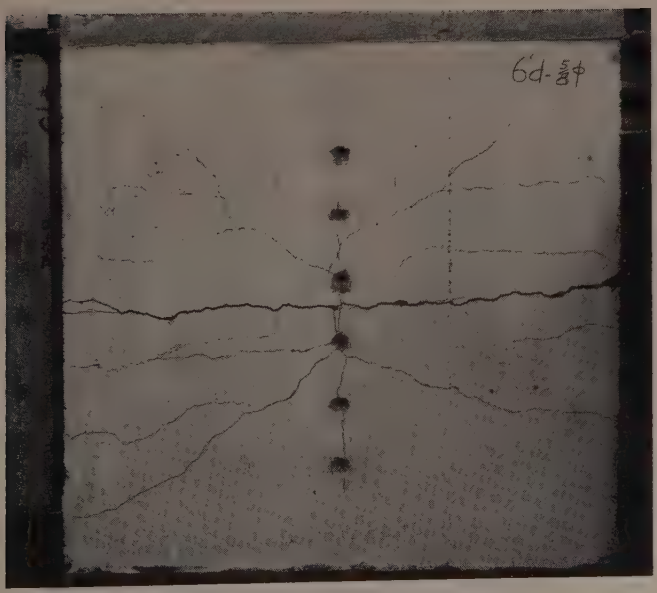
jd = the arm of the resisting moment

b = the periphery of a circle of diameter $D + 2d$. In the case of two tandem disks, with centers a distance a apart, $b = \pi(D + 2d) + 2a$.

It will be noted that the values of v developed are fairly uniform, varying from 166 to 216 lb. per sq. in., or from about 0.044 to 0.057



(a)



(b)

FIG. 32. VIEW OF SLABS OF SUPPLEMENTARY SERIES AFTER FAILURE

f_c', using as f_c' the average 28-day control cylinder strength for the group of 3800 lb. per sq. in. While the uniformity of these shearing stresses would indicate that the failure was directly related to the shearing stress, several other circumstances must also be recognized. In the first place the steel had yielded greatly and the concrete of the slab had developed many cracks before the final secondary failure by punching. Furthermore, the assumption of a conventional section a distance d outside the loaded area and the assumption that the shearing stress will be uniformly distributed over this section, in a one-way slab, may be far from correct. It seems logical that the shearing stress may be considered as governing the final failure, but that the absolute value of the stress should not be compared too closely with the shearing stresses developed in reinforced concrete members of dissimilar types, such as beams or footings.

IV. Discussion and Conclusions

23. *General Discussion.*—A brief comparison of the test results from the large slabs Nos. 1 and 2 and the smaller slabs of series B brings out some rather marked differences. Choosing those cases for this study in which the load was applied through a 6-in. disk, it is found that in six cases for slabs Nos. 1 and 2, a local punching failure developed at an average load of 81 550 lb., while in six cases of series B (6-in. disk on rubber and on plaster; 6-in. ring) the average value of the maximum load was 45 100 lb. This large difference is not confined to the ultimate load, for in both sets of tests it has been observed that the yield point of the reinforcing steel was reached at a load approximately 65 per cent of the ultimate load.

One obvious cause of a difference in maximum loads for the two types of slabs is found in the difference in slab thickness. The large slabs had a thickness of about 6.6 in. and an effective depth of 5.5 in., while the square slabs were 6 in. thick and had an effective depth of 5 in. or slightly less. A comparison of moments in the two types of slab shows that for the large slabs the moment per unit width under the load is $M_x = 0.333P$, $M_y = 0.265P$; for the small slabs,* $M_x = 0.370P$ and $M_y = 0.225P$. In the large slabs the reinforcement consisted of ½-in. square deformed bars spaced 4 in. apart in both directions; in the small slabs the ½-in. square deformed bars in the direction of the span were spaced 5 in. apart and those in the transverse direction, 10 in. apart. Although the moment M_y is less than M_x in these latter slabs it is more than half of M_x, so that M_y

*See V. P. Jensen, "Moments in Simple Span Bridge Slabs with Stiffened Edges," Bul. 315, Eng. Expt. Sta., Univ. of Ill., 1939, Tables 13, 14, 15.

should produce the greatest stress in the steel. Another consideration is that the steel was necessarily placed in two layers, that in the x direction in the lower layer and that in the y direction in the upper one. Hence the steel contributing to the resisting moment M_y acts at a disadvantage as to effective depth.

Comparing the value of $M_x = 0.333P$ for the large slab and $M_x = 0.370P$ for the small ones, and considering that the respective effective depths for the two are 5.5 in. and 5.0 in., it is evident that flexural stresses in the concrete at a given load might be roughly 35 per cent greater for the small slabs than for the large ones. Stresses in the reinforcement after cracking is general may be about 55 per cent greater in the x direction, and 135 per cent greater in the y direction, for the small slabs as compared to the large ones. This furnishes a partial explanation for the fact that the ultimate loads of the large slabs, loaded through 6-in. disks, averaged 81 per cent greater than those for the small slabs. Since the quality of both concrete and reinforcement was almost identical in the two series, the complete explanation of the difference in strength for the two types of slab is still lacking.

In comparing the long rectangular slab with the square one, it should be remembered that in the latter the range between maximum and average stress is much smaller than in the former. Thus if one considers the maximum stress at mid-span (σ_x across the y axis) the stress at the edges will be more than half the stress under the load for the square slab; in the slab 20 ft. wide, the stress at the edge is practically zero. Thus the square slab approaches the conditions of beam action (i.e. with uniform stress across the entire width) and a flexural type of failure might be expected instead of the punching failure which took place in most cases. The fact that failure was by bending and crushing rather than by punching in the supplementary tests mentioned in Section 20 indicates that the square slab is in a transitional zone between the beam, on one hand, and the very wide slab on the other. Apparently, the effect of a local yielding of the steel directly under the load is to produce a more uniform stress distribution over the slab width. Slight differences in ductility or in the character of the steel surface as related to bond resistance, are apparently enough to produce decided differences in the manner of failure.

Considerations of diagonal tension, as computed on the conventional section located a distance d (effective depth of slab) outside the limits of the loading disk, do not explain the definitely superior strength of the large slabs. Thus, while the shearing unit stresses

found from the ultimate loads of slabs 1 and 2 are about 310 lb. per sq. in. or 0.079 to 0.081 f'_c (where f'_c is the compressive strength of the concrete control cylinders), the corresponding values for the square slabs of series B vary from about 166 to 216 lb. per sq. in., or from 0.044 to 0.057 f'_c. It is doubtful whether these values of shearing unit stress, computed by a conventional method that is admittedly arbitrary, and applied to a portion of the slab which is quite generally cracked and distorted, are of any great significance. At best the failure is a secondary one, since the yield point of the steel in the direction of the span was passed long before the punching or shearing failures occurred. Another factor which should affect the values of shearing stress developed is the distribution of the shear around the conventional section. In the small slabs it seems likely that the shear at a distance d outside the loading disk was more variable than in the large slabs, hence the average values found for these slabs are probably less representative of the maximum shearing stress than in the large slabs.

24. *Conclusion.*—As a summary of the results of the tests the following general statements may be made:

(1) The experimental study of slabs of the type tested presented a number of difficult problems. The measurement of strains in the stage of loading before cracks had formed required very sensitive extensometers of the attached type; portable strain gages of the Berry or Whittemore type could only be used in the later stages of loading.

(2) No satisfactory solution to the problem of measuring reactions was found, although a number of devices were carefully tried out in preliminary tests. The elastic ring device used in the tests of slab No. 1 was first thought to be satisfactory, but the fact that it permitted a total deflection of 0.001 to 0.002 in. under a load of 1000 lb. per ring was shown by both tests and theory to destroy the accuracy of the reaction readings. As demonstrated in Appendix A, the distribution of slab reactions is extremely sensitive to slight deflections of the slab support.

(3) Deflection readings of the slab were fairly easy to duplicate, using portable deflectometers, and the measurements agreed very well with the computed values. However, it is well known that deflection curves are not particularly sensitive to considerable changes in moment or stress distribution, hence a good agreement between measured and theoretical deflections is principally an indication that the proper elastic constants were used in the computation.

(4) The materials used in the test slabs (except for a few supplementary tests of series B) were quite uniform. The yield point stress of the ½-in. square deformed bars was about 45 000 lb. per sq. in., and the average ultimate compressive strength of concrete control cylinders was about 3900 lb. per sq. in. The modulus of elasticity of the concrete of the slabs was evidently close to 4 500 000 lb. per sq. in., as determined from flexural tests.

(5) The strains as measured in the large rectangular slabs at low loads (4000 to 8000 lb.) have been compared with values computed by the ordinary theory, with Westergaard's correction applied to the region directly under the applied load. In a series of strain measurements taken with the load applied at the middle or quarter-point of the span, readings were taken on sections through the load point and in the direction of the span (x direction) or along the transverse center line (y direction). Except for a few unusually high strains directly under the load, the measured strains in the x direction were fairly close to the computed values, being generally a little high. The measured strains in the y direction along the x axis were consistently a considerable amount (perhaps 40 per cent) above the computed values. The measured strains along the transverse center line and in the y direction compared very well with the computed ones.

(6) The loads required to produce initial cracks in the large slabs varied from 8500 to 13 000 lb. The cracks at these loads were very fine and short. They were difficult to detect on the ordinary concrete surface, and closed up completely when the load was removed.

(7) Loads of 48 000 or 56 000 lb. applied at nine different points on each of the large slabs produced general cracking of the slab. Strain measurements indicated stresses in the reinforcement as great as 42 000 lb. per sq. in. at the 48 000-lb. load.

(8) Several local failures were produced in each large slab by choosing the load points at some distance apart, where there would be little effect due to the previous local yielding and fracture of the slab. These failures occurred at loads more than 50 per cent above the load that produced yield point stress in the reinforcement. The average load for six failures with the 6-in. loading disk was 81 550 lb., the load producing failure in the single test with a 2-in. disk was 60 500 lb. The thickness of these large slabs was 6.5 in., the effective depth about 5.5 in. The shearing unit stress on a conventional cylindrical section 5.5 in. outside the loading disk was about 310 lb. per sq. in.

(9) The secondary failures mentioned in the foregoing developed after the steel had yielded and many cracks had formed in the concrete under the load. Failure occurred by punching out a plug or truncated cone of concrete, in all cases except those in which the load was very close to one of the supports.

(10) In the tests of series A on slabs about 5 ft. square, the strains at low loads were not greatly affected by the size of bearing plate used or by the bedding material, sponge rubber or plaster of Paris, used between steel plate and slab. There was a tendency for the strains to decrease slightly for the larger bearing plates, as for example, the 14-in. disk, the 14-in. ring and the tandem 6-in. disks.

(11) At the higher loads of series B, the strains in the reinforcing steel of the 5-ft. square slabs were decidedly less for the 14-in. bearing disk than for the 2-, 6- and 10-in. disks. The 14-in. ring and tandem 6-in. disks also produced relatively low steel strains.

(12) The ultimate strengths obtained in series B with bearing disks and rings of the same diameter were practically identical. The results obtained with tandem 2-in. disks with centers 3 in. apart were about equal to those for the 6-in. disk; those for the tandem 6-in. disks, with centers 12 in. apart, were about equal to those for the 14-in. disk. The ultimate strength increased in a fairly consistent proportion to the diameter of disk.

(13) Failure occurred in the slabs of series B by punching out a truncated cone of concrete directly under the loading disk. However, such failures occurred at loads 50 per cent or more above the load producing yield point stress in the steel, and after a large number of cracks had formed in the slab. In the slabs with 14-in. loading disks there was evident a tendency for the slab to fail as a beam, and, in supplementary tests with a different type of reinforcing steel, failures of a definitely flexural type took place, with the development of transverse cracks, crushing of the concrete, and gradual yielding of the slab.

(14) The shearing unit stress at which punching failure of the slabs occurred has been studied. However, since such secondary failures came so long after the yield point of the steel had been reached and cracking of the slab had become general, the shearing stresses are probably not significant. The shearing stresses, varying from 0.044 to 0.057 f'_c for the small slabs, and 0.079 to 0.081 f'_c for the large ones, when computed on the conventional section a distance d outside the loading disk, are undoubtedly smaller than would be expected with a larger steel percentage or stronger steel.

(15) A comparison of measured and computed strains in the reinforcement of the various slabs tested indicates a fairly satisfactory,

though not complete, agreement. In the tests of the two large slabs, the highest of the measured strains at any point were about equal to the computed values, except directly under the load point, where the ratio of measured to calculated values generally did not exceed 0.80. This may be explained by large strain variations within the gage length, by possible bond slip, and by the fact that the computed values involve some inconsistencies in assumed conditions. However, the tests of the 5-ft. square slabs produced measured strains that compared very closely with the calculated ones, even though the strains considered were those directly under the load point. The results of the two sets of comparisons indicate a fairly satisfactory check between analysis and tests with regard to steel strains. This is important, since primary failure in a reinforced concrete slab usually occurs through yielding of the tensile steel, although the foregoing tests also showed a fairly satisfactory margin of safety after the yield point stress in the reinforcement had been reached.

APPENDIX A

The Effect of Flexibility of the Reaction Weighing Rings on the Measured Reactions for the Test Slab

In order to study the effect of the flexibility of the rings used to measure the reactions at one edge of test slab No. 1 an approximate study was made of the slab shown in Fig. 33. The two short edges of this slab and one of the long edges are assumed simply supported, and the other long edge is assumed to rest on a series of springs having a load-deflection relation the same as that of the reaction rings used in the central portion of slab No. 1. It is believed that for a rectangular slab of the relative dimensions indicated in Fig. 33 the significant effects are very nearly the same as in the test slab which lacks supports on the short sides, but is only 3 times as long as it is wide.

The calculations were made by means of the distribution procedure described in a previous report* of this investigation. In the calculations the value of N for the slab was taken as 112 000 000 in. lb., as determined by a deflection test on the slab for which the reactions were measured. The value of Poisson's ratio, μ, was assumed to be 0.15, and the rigidity of the flexible spring supports was assumed to correspond to an elastic deflection of the reaction rings of 0.002 in. per 1000 lb. load, with the rings spaced 6 in. apart. That is, the spring constant for the flexible support was taken as 83 333 lb. per sq. in., which means, for example, that a distributed load of 83.33 lb. per lineal inch of the support, or 500 lb. on a 6-in. length, would produce a deflection of 0.001 in.

For a load P at the center of the slab, the reaction of the slab along the support OB, if the support does not deflect, is given by the equation:

$$R = \sum_{1,3,5,\cdots} R_n \left(\frac{2P}{a} \sin \frac{n\pi}{2} \right) \sin \frac{n\pi x}{a} \qquad \text{(A1)}$$

where

$$R_n = C + (1 - \mu)\, m_x \qquad \text{(A2)}$$

as given in Section 8 of Appendix B of Bulletin 304. Values of C and m_x are given in Tables 6 and 8, respectively, of Bulletin 304, for various

*N. M. Newmark, "A Distribution Procedure for the Analysis of Slabs Continuous over Flexible Beams," Bul. 304, Univ. of Ill., Eng. Expt. Sta., 1938.

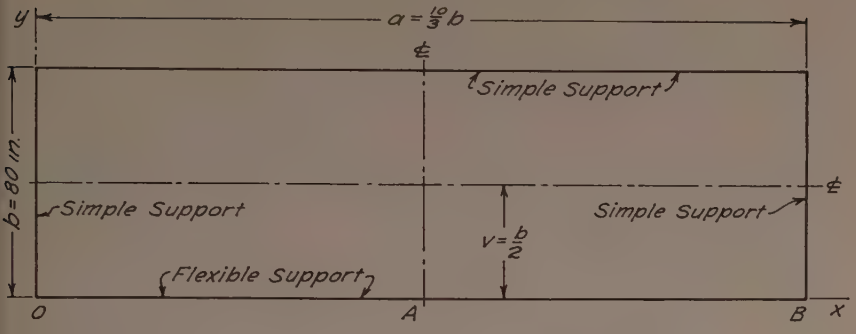


FIG. 33. SLAB HAVING FLEXIBLE SUPPORT UNDER ONE EDGE

values of b/s or of $n\,b/a$. For a load at the center of the slab the numerical values are to be taken from the column headed $v/b = 0.5$.

The resistance of the slab to a sine curve of deflection of the edge OB is defined as the maximum ordinate of the sine curve of reaction required to produce a corresponding sine curve of deflection of unit maximum ordinate, and is represented by the symbol T' where T' varies with the number of waves, n, in the sine curve of deflection or reaction. The magnitude of T' may be derived by the method shown in Fig. 7 of Bulletin 304, and is as follows:

$$T' = T - \frac{Q^2}{S}(1 - 2kq + q^2)$$

or

$$T' = C'_T \frac{N}{b^3} \qquad \text{(A3)}$$

where C'_T is a numerical constant depending on b/s or $n\,b/a$. Numerical values of C'_T may be calculated from the values of T, Q, S, k and q given in Tables 1 and 2 of Bulletin 304.

The stiffness of the flexible support OB, defined in the same manner as the stiffness of the edge OB of the slab, is denoted by J. The numerical value of J, as has been mentioned previously, is 83 333 lb. per sq. in. for all values of n, in this particular problem.

When the flexible support is permitted to deflect, the resultant reaction on the flexible support becomes $\overline{R}$ where $\overline{R}$ is given by the equation:

$$\overline{R} = \sum_{1,3,5} \overline{R}_n \left(\frac{2P}{a} \sin \frac{n\pi}{2} \right) \sin \frac{n\pi x}{a} \qquad \text{(A4)}$$

TABLE 6
MAXIMUM REACTIONS AT A PER UNIT LENGTH
Load at center of slab (see Fig. 33)

n	b/s	j_n	Distributed Reaction For Rigid Support R_n	Distributed Reaction For Flexible Supports $\bar{R}_n$
1	0.3	0.9954	0.4887	0.4865
3	0.9	0.9252	0.3523	0.3259
5	1.5	0.7310	0.1863	0.1362
7	2.1	0.4977	0.0885	0.0440
9	2.7	0.3180	0.0403	0.0128
11	3.3	0.2034	0.0179	0.0036
13	3.9	0.1340	0.0079	0.0011
15	4.5	0.0915	0.0035	0.0003
17	5.1	0.0647	0.0015	0.0001
19	5.7	0.0472	0.0006	0.0000
21	6.3	0.0354	0.0003	0.0000
23	6.9	0.0272	0.0001	0.0000
Sum			$R_A = 1.1879$	$\bar{R}_A = 1.0105$
Relative values, per cent			$R_A = 100.0$	$\bar{R}_A = 85.1$

in which $$\bar{R}_n = \frac{J}{J + T'} R_n = j_n R_n \qquad \text{(A5)}$$

for each value of n.

With the numerical values mentioned in the foregoing, the value of j_n may be expressed as follows:

$$j_n = \frac{380.95}{380.95 + C'_T} \qquad \text{(A6)}$$

Numerical values of j_n, R_n, and $\bar{R}_n$ are given in Table 6.

If the magnitude of the load P is taken numerically equal to $a/2$, the distributed reaction per unit length at A, for a rigid support, is given by the expression:

$$R_A = \sum_{1,3,5} R_n$$

and for a flexible support,

$$\bar{R}_A = \sum_{1,3,5,\cdots} \bar{R}_n = \sum_{1,3,5,\cdots} j_n R_n$$

The calculations for R_A and $\overline{R}_A$ at A due to a load at the center of the slab are summarized in Table 6. It appears that, for a support of the flexibility assumed, the reaction per unit length at A is 85.1 per cent of the corresponding reaction for a rigid support. Similar calculations for the support OB, assumed twice as stiff, or with $J = 166{,}667$ lb. per sq. in., indicate a reaction per unit length at A of 90.1 per cent of the corresponding reaction for a rigid support.

For a load directly over the edge of the slab at A the value of R_n is found to be 1.0 for all values of n. This indicates that the reaction per unit length for a rigid support is infinite at point A for such a loading. One may, however, compute the reaction due to a load uniformly distributed over a 6-in. length, by means of the methods described in Bulletin 304. The result is merely the load divided by the distance 6 in. If P is taken numerically equal to $a/2$, this corresponds to a value of R_A of 22.222. The value of $\overline{R}_A$ may be computed. However, to a very close approximation the value of $\overline{R}_A$ is the sum of the values of j_n since R_n is very close to unity for the loading considered. It is necessary to take a larger number of terms than are recorded in Table 6 in order to obtain the final results. With a fairly accurate estimate of the effect of the terms for large values of n, one arrives at the result, for a load at A, that $\overline{R}_A = 4.21$. This is only 18.9 per cent of the corresponding value for rigid supports. Corresponding calculations for J twice as great indicate a value of 23.9 per cent for the ratio of the reaction per unit length at A with flexible supports to the reaction with rigid supports. It should be remembered that these calculations involve additional assumptions beyond those made for a load at the center of the slab, and are therefore less reliable.

These results may be taken as an indication of the influence of the flexibility of the reaction measuring devices on the measured reactions.

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